ACTIVITY BOOK

INTEGRAL CALCULUS

AMARGA
BEGORNIA
LAGUILLES
URBANO

POWER FORMULA

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. The power rule for integration provides us with a formula that allows us to integrate any function that can be written as a power of x. *Answer:* ***True***
2. The power rule of integration can be applied when n = -1. *Answer:* ***False***
3. The power rule says that: ∫ xn dx = (xn+1) / (n+1) + C (where n ≠ -1). *Answer:* ***True***
4. The integral of ∫ 1 / (x^3) dx using power rule is $\frac{x^{-2}}{-2} + C$. *Answer:* ***True***
5. We cannot integrate polynomials, negative exponents, and radicals using the power rule. *Answer:* ***False***

EVALUATE

Evaluate the following:

6. Evaluate $\int 7x^6 dx$ *Answer:* $x^7 + C$
7. Evaluate $\int \left(3x^2 - \sqrt{9x} + 7\right)dx$ *Answer:* $x^3 - \frac{2}{3}x\sqrt{9x} + 7x + C$
8. Evaluate $\int (7x^2 - 8x^7)dx$ *Answer:* $x^7 - x^6 + C$
9. Evaluate $\int (8x + 4)(2x + 5)dx$ *Answer:* $16\frac{x^3}{3} + 24x^2 + 20x + C$
10. Evaluate $\int \sqrt[4]{x^7}$ *Answer:* $\frac{4}{11}\sqrt[4]{x^{11}} + C$

EXPONENTIAL FUNCTION

TRUE OR FALSE

Indicate whether the following statements are true or false.

1.The derivative of f(x)= 5^{x} is an exponential function. *Answer.* ***True***
2. All exponential functions have vertical asymptote. *Answer.* ***False***
3. The range of an exponential function is a set of all positive real numbers. *Answer.* ***True***
4. The y-intercept of all the exponential functions is 1. *Answer.* ***False***
5. The Domain name of an exponential function is a set of all positive real numbers. *Answer.* ***True***

INTEGRATE

Integrate the following:

6. Integrate $\int(2\text{-}3e^{x})dx$ Answer. $2x$ - $3e^{x}$+C

7. Integrate $\int e^{x}(1+2e^{x})^{4}dx$ Answer. $(1/10)(1+2e^{x})^{5}$+ C

8. Integrate $\int(e^{4x}\text{-}\ e^{-4x})^{2}dx$ Answer. $\frac{e^{8x}}{8}$ - 2x - $\frac{e^{-8x}}{8}$+ C

9. Integrate $\int e^{x}(1-e^{x})(1+e^{x})^{10}dx$ Answer. $(2/11)(1+e^{x})^{11}$- $(1/12)(1+e^{x})^{12}$+ C

10. Integrate $\int e^{5x}(\frac{e^{2x}}{7}+\frac{3}{e^{3x}})dx$ Answer. $(1/49)e^{7x}$+$(3/2)e^{2x}$+ C

SPECIAL COMBINATION

MATCHING TYPE

Match Column A with Column B.

COLUMN A	COLUMN B
1. $\int \sec u \tan u \, du$	a. $-\cot u + c$
2. $\int \csc^2 u \, du$	b. $\tan u + c$.
3. $\int \sec^2 u \, du$	c. $-\csc u + c$
4. $\int \csc u \cot u \, du$	d. $\sec u + c$
5. $-\cot u + c$	e. $\int \csc^2 u \, du$

INTEGRATE

Integrate the following:

6. $\int (\csc 3y + \cot 3y)^2 \, dy$

7. $\int \tan^2 y \sec y \, dy$

8. $\int 5 \sec 4\theta \tan 4\theta \, d\theta$

9. $\int \sec^4 3x \tan^3 3x \, dx$

10. $\int \cot^4 y \, dy$

TRIGONOMETRIC FUNCTIONS

ANSWER

Answer the following questions.

1. The integration of a function f(x) is given by F(x) and it is represented by what? — ANSWER: *∫f(x)dx = F(x) + C*
2. What will you use to integrate this given trigonometric function $cos^2 x = \left(\frac{1+cos2x}{2}\right)$? — ANSWER: *trigonometric* $identity$
3. According to the properties of integration, does the integral of sum of two functions equal to the sum of integrals of the given functions? — ANSWER: *Yes*
4. What is the integral of the function sinx? — ANSWER: *-cscx+C*
5. What is the integral of the function cosx? — ANSWER: sinx+C

EVALUATE

Evaluate the following:

6. Evaluate $\int 6sec\ 3x\ tan\ x\ 3x\ dx$ — ANSWER: *2sec3x+C*
7. Evaluate $\int cot^3 ax\ csc^2 ax\ dx$ — ANSWER: $-\frac{1}{4a}\left(cot^4 ax\right) + C$
8. Evaluate $\int 7cos^2 9xdx$ — ANSWER: $\frac{7}{2}x + \frac{7}{36}sin18x + C$
9. Evaluate $\int (csc6x + cot6x)^2 dx$ — ANSWER: $-\frac{2}{6}cot6x - \frac{2}{6}csc6x - x + C$
10. Evaluate $\int tan7xdx$ — $ANSWER$: $-\frac{1}{7}lcos(7x) + C$

HYPERBOLIC FUNCTIONS

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. Hyperbolic function are defined in terms of exponential functions. Ans: True
2. The properties of hyperbolic function is analogous to the trigonometric function. Ans: True
3. Hyperbolic function cannot be used to model catenaries. Ans: False, (can be)
4. The hyperbolic function $\int \coth u \, du$ is equal to $-\ln[\operatorname{sech} u] + C$. Ans: True
5. The hyperbolic function $\int \cosh u \, du$ is equal to $-\ln[\operatorname{csch} u] + C$. Ans: False, $\sinh u + C$

INTEGRATE

Integrate the following:

6. $\int \frac{\cosh x}{\cosh^2 x} dx$ Ans: $\arctan(\sinh(x)) + C$

7. $\int_0^{\ln 4} 4e^x \sinh x \, dx$ Ans: $15 - \ln 16$ or 12.227

8. $\int e^{3x} \sinh x \, dx$ Ans: $\frac{e^{4x} - 2e^{2x}}{8} + C$

9. $\int \frac{\cosh x}{6+5\sinh x} dx$ Ans: $\frac{1}{5}\ln(6 + 5\sinh x) + C$

10. $\int 6e^x \sinh x \, dx$ Ans: $\frac{3}{2}e^{2x} - 3x + C$

INVERSE TRIGONOMETRIC FUNCTIONS

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. Unless the domains are restricted, trigonometric functions are not one to one. Answer: *TRUE*
2. There are six inverse trigonometric functions. Answer: *TRUE*
3. The rule on integration only mentions three integration formulas. Answer: *TRUE*
4. In the list of integrals of inverse trigonometric functions, there is a formula for each inverse hyperbolic integration formula. Answer: *TRUE*
5. The constant *a* is assumed to be zero in all formulas, and C is the constant of integration. Answer: *FALSE*

EVALUATE

Evaluate the following:

6. Find the values of $\sin(\cos^{-1} 5/6)$ Answer: $\frac{\sqrt{11}}{6}$
7. Find the values of $\cos(\tan^{-1} 2/3)$ Answer: $\sqrt{\frac{13}{9}}$
8. Evaluate $\int \frac{xdx}{\sqrt{16-81x^4}}$ Answer: $\frac{1}{8}sin^{-1}\left(\frac{9x^2}{4}\right) + C$
9. Evaluate $\int \frac{dx}{144\sqrt{x}+x\sqrt{x}}$ Answer: $\frac{2}{12}tan^{-1}\frac{\sqrt{x}}{12} + C$
10. Evaluate $\int \frac{10}{x\sqrt{8x^2-1}}$ Answer: $10sec^{-1}(4x) + C$

INTEGRALS OF TERMS INVOLVING U2+ A2 AND A2-U2

EVALUATE

Evaluate the following:

1. Evaluate $\int \frac{3}{x\sqrt{x^2-9}}dx$ — Ans: $asec(\frac{|x|}{3}) + C$

2. Evaluate $\int \frac{e^x}{\sqrt{7-e^{2x}}}dx$ — Ans: $arcsin(\frac{\sqrt{7e^x}}{7}) + C$

3. Evaluate $\int \frac{1}{\sqrt{x^2-a^2}}dx$ — Ans: $In\ (|x + \sqrt{x^2 - a^2}|)$ + C

4. Evaluate $\int \frac{sin\ x}{\sqrt{2-cos\ ^2x}}dx$ — Ans: $-\ arcsin(\frac{\sqrt{2\ cos\ (x)}}{2}) + C$

5. Evaluate $\int \frac{1}{\sqrt{1-4x^2}}dx$ — Ans: $\frac{arcsin\ (2x)}{2} + C$

INTEGRATE

Integrate the following:

6. Integrate $\int \frac{5}{36-x^2}dx$ — Ans: $\frac{5}{12}In\ |\frac{x+6}{x-6}| + C$

7. Integrate $\int \frac{x}{\sqrt{x^4+16}}dx$ — Ans: $\frac{1}{2}In\ |x^2 + \sqrt{x^4 + 16}$ + C

8. Integrate $\int \frac{2}{\sqrt{49-x^2}}dx$ — Ans: $2\ arcsin(\frac{x}{7})$ + C

9. Integrate $\int \frac{4x+5}{x^2+4}dx$ — Ans: $2\ In\ (x^2 + 4)$ + $\frac{5}{12}arctan\ (\frac{x}{2})$ + C

10. Integrate $\int \frac{7}{16+x^2}dx$ — Ans: $\frac{7}{4}arctan\ (\frac{x}{4})$ + C

INTEGRATION OF POWERS & PRODUCTS OF SIN & COS

EVALUATE

Evaluate the following:

1. $\int x^2 \cos x \ dx$
2. $\int \sin 3\theta \cos 4\theta \ d\theta$
3. $\int \sin^3 y \cos^2 y \ dy$
4. $\int x \ \sin 3x \ dx$
5. $\int (\sin y + \cos y)^2 dy$

ENUMERATE

Enumerate the derived form of the following sine/cosine product identity

6-7. $\cos A \cos B = 12[\cos(A-B) + \cos(A+B)]$ Ans. $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

8-9. $\sin A \sin B = 12[\cos(A-B) - \cos(A+B)]$ Ans. $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

10. $\sin A \cos B = 12[\sin(A-B) + \sin(A+B)]$ Ans. $\sin(A+B) = \sin A \cos B - \cos A \sin B$

$\sin(A-B) = \sin A \cos B + \cos A \sin B$

INTEGRATION BY PARTS

ANSWER

Answer the following questions:

1. In $\int sec^2\, x\, dx$ what will be the value of u? Ans: $\sec x$
2. What is the formula of integration by parts? Ans: $\int udv = uv - \int vdu$
3. ILATE is used in determining and prioritizing u. (True or False) Ans: True
4. In the ILATE, the I mean Integration. (True or False) Ans: False, (Inverse function)
5. In the ILATE, the E means exponential. (True or False) Ans: True

INTEGRATE

Integrate the following:

6. $\int_0^{\pi} a^2 \cos(6a)\, da$ Ans: $\frac{\pi}{18}$
7. $\int_0^5 (3 + 5x)e^{\frac{1}{3}x}\, dx$ Ans: 242.4851
8. $\int x^4 \ln x\, dx$ Ans: $\frac{x^5(5\ln(x)-1)}{25} + C$
9. $\int e^{6x} \cos(e^{2x})dx$ Ans: $\frac{(e^{4x}-2)\sin(e^{2x})}{2} + e^{2x}\cos(e^{2x}) + C$
10. $\int e^{4x} \cos(2x)dx$ Ans: $\frac{e^{4x}(\sin(2x)+2\cos(2x))}{10} + C$

INTEGRATION BY TRIGONOMETRIC SUBSTITUTION

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. The three common trigonometric substitution are the restricted sine, restricted tangent and restricted secant. Ans: True

2. Typically trigonometric substitutions are used for problems that involve radical expression s? Ans: True

3. In integration by substitution, when $\sqrt{a^2+u^2}$ the value of u is $a\tan\theta$. Ans: True

4. In integration by substitutions, when $\sqrt{a^2-u^2}$ the value of du is $a\sin^2\theta\ d\theta$. Ans: False, $a\cos\theta\ d\theta$

5. In integration by substitutions, when $\sqrt{u^2-a^2}$ the value of du is $a\cos^2\theta\ d\theta$. Ans: False, $a\sec\theta\tan\theta\ d\theta$

INTEGRATE

Integrate the following:

6. $\int \frac{\sqrt{16-x^2}}{x^2}dx$ Ans: $-\frac{\sqrt{16-x^2}}{x}-\sin^{-1}\left(\frac{x}{4}\right)+C$

7. $\int \sqrt{1-6y^2}\,dy$ Ans: $\frac{1}{2\sqrt{6}}\left[\sin^{-1}(\sqrt{6}y)+\sqrt{6}y\sqrt{1-6y^2}\right]+C$

8. $\int \cos x\sqrt{4+25\sin^2 x\ dx}$ Ans: $=\frac{2\ln(|\sqrt{25\,sin^2\,x+4}+5\sin x|)}{5}+\sin x\sqrt{\frac{25\sin^2(x)}{4}+1}+C$

9. $\int_1^3 2x^5\sqrt{4+16x^2}dx$ Ans: $=2546.1589$

10. $\int_{-6}^{-4}\frac{3}{x^3\sqrt{x^2-16}}dx$ Ans: $=-0.015301$

INTEGRATION BY RATIONAL FUNCTION

INTEGRATE

Integrate the following:

1. $\displaystyle\int \frac{4x+3}{(x-2)^2(x+3)}dx$ Ans. $\frac{9}{25}\ln|x-2| - \frac{11}{5(x-2)} - \frac{9}{25}\ln|x+3| + C$
2. $\displaystyle\int \frac{4}{(x^2-4)^2}dx$ Ans. $4\left(\frac{1}{32}\ln|x+2| - \frac{1}{16(x+2)} - \frac{1}{32}\ln|x-2| - \frac{1}{16(x-2)}\right) + C$
3. $\displaystyle\int \frac{3x}{(x+1)^2(x-3)}dx$ Ans. $3\left(-\frac{3}{16}\ln|x+1| - \frac{1}{4(x+1)} + \frac{3}{16}\ln|x-3|\right) + C$
4. $\displaystyle\int \frac{2}{(x+2)(x-4)}dx$ Ans. $2\left(-\frac{1}{6}\ln|x+2| + \frac{1}{6}\ln|x-4|\right) + C$
5. $\displaystyle\int \frac{3}{x^2+5x+6}dx$ Ans. $3(\ln|x+2| - \ln|x+3|) + C$

TRUE OR FALSE

Indicate whether the following statements are true or false.

6. At-times, integration of a proper rational function leads to a logarithm. Ans. True
7. The partial fraction decomposition of $\frac{2x+3}{x^2}$ is $\frac{2}{x} + \frac{3}{x^2}$ Ans. True
8. A polynomial function is a rational function. Ans. True
9. Partial fraction decomposition is most effective in integrating rational functions. Ans. True
10. We can use algebraic techniques to integrate the ratio of polynomials. Ans. True

WALLIS FORMULA

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. $\frac{\pi}{5}$ is the α value if the m and n are both positive even? Ans: False, $\frac{\pi}{2}$
2. 1 is the α value if the m and n are either or both odd? Ans: True
3. We use Walli's formula if the upper limit is $\frac{\pi}{2}$. Ans: True
4. We use Walli's formula if the limit is $0\ to\ \frac{\pi}{10}$. Ans: False, (0 to $\frac{\pi}{2}$)
5. In Walli's formula, ξ is known as skip factorial. Ans: True

INTEGRATE

Integrate the following:

6. $\int_0^{\frac{\pi}{2}} sin^9 v\ cos^2\ v\ dv$ Ans: $\frac{128}{3465}$
7. $\int_0^{\frac{\pi}{2}} cos^8 a\ sin^5 a\ da$ Ans: $\frac{8}{1287}$
8. $\int_0^{\frac{\pi}{10}} \sin^6(5x)\cos^4(5x)\ dx$ Ans: $\frac{3\pi}{2560}$
9. $\int_0^{a} x^2(a^2 - x^2)^{\frac{3}{2}} dx$ Ans: $\frac{\pi a^6}{32}$
10. $\int_0^{\frac{\pi}{6}} \sin^4(3x)\cos^5(3x)\ dx$ Ans: $\frac{8}{945}$

ODD AND EVEN FUNCTION

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. Odd Function is unchanged when rotated 180 degree about the origin. Ans.True
2. The definite integral of an odd function is always 0. Ans.True
3. If f is odd then $\int_{-a}^{a} f(x)dx = 0$ Ans.True
4. The definite integral of an odd function between -a and a is always 0. Ans.True
5. When two odd functions are multiplied, the combined function is also an odd function. Ans.False

INTEGRATE

Integrate the following:

6. $\int_{-\pi}^{\pi} - 5\, sin\, xdx$ Ans. 0
7. $\int_{-2}^{2} xdx$ Ans. 0
8. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 sin\, x}{1+x^6} dx$ Ans. 0
9. Find $\int_{3}^{6} f(x)dx$ if $f(x)$is odd and $\int_{3}^{6} f(x)dx = 10$ Ans. 0
10. $\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}}$ 9 sin(x)dx Ans. 0

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. An even function is a function such that f(x)=f(−x) where the value remains unchanged if the sign of the independent variable is reversed. ANSWER: *TRUE*
2. The graph of an even function is symmetric about the y-axis. ANSWER: *TRUE*
3. An even function always has an even anti-derivative. ANSWER: *FALSE*
4. Sin x is an even function. ANSWER: *FALSE*
5. An even function must pass through the origin. ANSWER: *FALSE*

INTEGRATE

Integrate the following:

6. Evaluate $\int_{-4}^{4} x^3 + 1\,dx$ ANSWER: *272*

7. Evaluate $\int_{-2}^{2} (x^2 + 2)\,dx$ ANSWER: *40/3=13.33*

8. Evaluate $\int_{-2}^{2} \left(3x^4 - \frac{1}{2}x^3\right)dx$ ANSWER: *1458/5=291.6*

9. Evaluate $\int_{-2}^{2} (3x^3 - 2x^2 + 3)\,dx$ ANSWER: *1028/3=342.67*

10. Evaluate $\int_{-2}^{2} x^4 + x^3\,dx$ ANSWER: *312/7=44.57*

AREA BOUNDED BY TWO CURVES

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. One function value is always greater than the other. ANSWER: *TRUE*
2. The area between two curves can be composite. ANSWER: *TRUE*
3. The area between two curves is the area that falls in between two intersecting curves. ANSWER: *TRUE*
4. Integration can't be used to find the area under two curves. ANSWER: *FALSE*
5. It has two functions f(x) and g(x). ANSWER: *TRUE*

PROBLEM-SOLVING

Find the area of the following:

6. Find the area enclosed by the following two functions: $y=x^2$, $y=4x-x^2$. ANSWER: *8/3*
7. Find the area between y=4x^2 and y=5-x^2 over the interval from -1 to 1. ANSWER: 20/3
8. Find the area between the graphs of f(x) and g(x)=x-1 [-2,1]. ANSWER: *16.5*
9. Find the area at the region bounded by $y=x^2+2x-6$ and y= 3x. ANSWER: *125/6*
10. Find the area between $y=3x^2+5$ and y= -x+3 over the interval from 0 to 2. ANSWER: *14*

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. The formula that we use in solving the area (left and right curve) is $A = \int_{y1}^{y2} \left(x_T - x_B\right) dy$. Ans: True
2. In finding the upper and lower function is difficult without a graph. Ans: True
3. The total area under any of the curves is equal to 1. Ans. True
4. The curve of a normal distribution extends indefinitely at the tails. Ans. True
5. The area under the normal curves is 100. Ans. False

PROBLEM-SOLVING

Find the area of the following:

6. Calculate the area of the region bounded by $x = 4 + y^2, x = 3 - y^2,\ y = -1$ and $y = 2$. Ans: 9
7. Determine the area of the region bounded by $x = -y^2 + 5$ and $x = (y - 1)^2$. Ans: 9
8. Determine the area of the region bounded by $x = y^2 - 3y - 7$ and $x = y + 5$. Ans: $\frac{256}{3}$ $or\ 85.3333333$
9. Determine the area of the region bounded by $x = y^2 + 2, x = 6, y = -4$ and $y = 4$. Ans: 32
10. Determine the area of the region bounded by $x = e^{1+2y}, x = e^{1-y}, y = -1$ and $y = 2$. Ans: 73.9926

PROBLEM-SOLVING

Find the area of the following:

1. Find the area inside the graph of $r = 7 + 3\ \cos\theta$ and to the left of the y-axis.

Ans. $42.0376\ sq.\ \ units$

2. The area bounded by the loop of the curve $4y^2 = x^2(4 - x^2)$ is Ans. $\frac{16}{3} sq. units$

3. The area of the loop of the curve, $ay^2 = x^2(a - x)$ is Ans. $\frac{8a^2}{16} sq.\ \ units$

4. Find the area that is inside $r = 3 + 3\sin\theta$ and outside $r = 2$ Ans. $33.7074 sq.\ \ units$

5. Find the area inside the inner loop of $r = 3 - 8\ \cos\theta$ Ans. $15.2695\ sq.\ \ units$

TRUE OR FALSE

Indicate whether the following statements are true or false.

6. A polar curve is a shape constructed using the polar coordinate system. Ans. True

7. Each point in the polar coordinate system is given by *(r, σ)*(*r*,σ), where *r* is the distance from the pole (origin) to the point, and σ is the counterclockwise angle that is made with the point, pole, and the positive *x*-axis. Ans. False, θ

8. Certain relations that are not functions in Cartesian form can be expressed as functions in polar form. Ans. True

9. The area enclosed by a polar curve cannot be computed with integration. Ans. False

10. Every point in the Cartesian plane has three values associated with it. Ans. False, two

ARC LENGTH

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. The length of an arc is the same thing as the measure of the arc. Ans. False
2. The techniques we use to find arc length can be extended to find the surface area of a surface of revolution Ans. True
3. To find the surface area of the band, we need to find the lateral surface area, S of the frustum.

   Ans. True
4. Surface area is the total area of the outer layer of an object. Ans. True
5. If the curve represents the path of a moving object, the length of the curve between two points may be the distance traveled by the object between two times. Ans. True

PROBLEM-SOLVING

Find the area of the following:

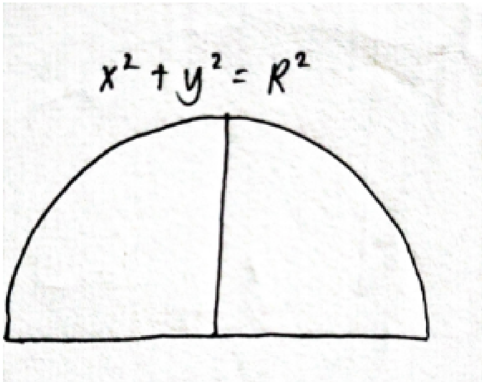
6. Find the length of the line segment given by the equation $y = 8x + 4$

   from $x = 2\ to\ x = 5$. Ans: $24.18677\ units$
7. Find the length of the curve $r = 3(1 - \cos\theta)$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$. Ans: $-3.5147\ units$
8. Determine the length of $y = 2 + 4x^{\frac{3}{2}}$, $\quad 0 \leq x \leq 3$. Ans: $21.0554\ units$
9. Determine the length of the curve $x = \frac{1}{3}(y^2 + 4)^{\frac{3}{2}}$, $1 \leq y \leq 2$. Ans: $3.33333333333\ units$
10. Find the length of the curve $x = \frac{1}{2}\sqrt{y}\,(y - 2)$ from $y = 0$ to $y = 3$. Ans: $3.4641\ units$

PAPPUS THEOREM PROBLEM-SOLVING

Find the area of the following:

1. Graph the region R bounded by $y = 4x - x^2$ and the x- axis, and find the centroid of R
2. Solve for the centroid of the semi-disc shown in the figure below. Then find the volume of the sphere using Pappus' Theorem.



3. Solve for the centroid of the region R bounded by $y = \sin x$ and the x-axis, $0 \le x \le \pi$.
4. Graph the region R bounded by the curves $y = \dfrac{1}{2}x^2$ and $x = \dfrac{1}{2}y^2$ and solve for the centroid of R.
5. Graph the region R bounded by the curves $y = x^2$ and $y = x + 2$ and solve for the centroid of R.

TRUE OR FALSE

Indicate whether the following statements are true or false.

6. Pappus' Theorem is usually attributed to Pappus of Alexandria Ans. True
7. If a plane area is rotated about an axis in its plane, but which does not cross the area, the volume swept out equals the area times the distance moved by the centroid.

   Ans. True

8. If a plane curve is rotated about an axis in its plane, but does not cross the curve, the area swept out equals the length times the distance moved by the centroid.

   Ans. True

9. The first theorem of Pappus states that the surface area A of a surface of revolution generated by the revolution of a curve about an external axis is equal to the product of the arc length s of the generating curve and the distance d_1 traveled by the curve's geometric centroid $\overline{x}$, Ans. False, SA(S)
10. The centroid of a uniform semicircle of radius R is 0.48 R. Ans. False, 0.42 R

PAPPUS THEOREM ||

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. A solid of revolution must have cross-sections which are circular(i.e. either disk or washer)with the axis of revolution running through the center of the disk . A pyramid does not have any circular cross-sections, so it cannot be a solid of revolution. Ans. False
2. The shell method always produces shells parallel to the y-axis. Ans. False
3. The disk/washer method may be used by integration with respect to y. Ans. False
4. The rotation of the function about the axis creates a cylindrical shape parallel to the axis. Ans. True
5. If the disk/washer are perpendicular to the axis of revolution x must be used and if the shells are parallel to the axis of revolution y must be used. Ans. True

PROBLEM-SOLVING

Find the area of the following:

6. Determine the volume generated by rotating the curve $16x^2 + 4y^2 = 11$ about the line $4x + 2y = 25$. Ans: $388.42\ cu.\ units$
7. Determine the volume of the solid obtained by rotating the region bounded by $x = (y - 4)2$, the x-axis and the y-axis about the x-axis. Ans: $134.0412\ cu.\ units$
8. Determine the volume of the solid obtained by rotating the region bounded by $y = \sqrt[2]{x}$, $x = 9$ and the x-axis about the x-axis. Ans: $127.2345\ cu.\ units$
9. Determine the volume of the solid obtained by rotating the region bounded by $y = 9x$ and $y = x3$ about the y-axis. For this problem assume that $x \geq 0$. Ans:$-\ 203.5752\ cu. units$
10. Determine the volume of the solid obtained by rotating the region bounded by $y = 3x + 2$, $y = 1$ and $x = 2$ about the line $y = 10$. Ans: $\frac{980\pi}{9}\ cu.\ units$

CIRCULAR DISK METHOD

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. What is the formula for horizontal strip? ANSWER: $A=\int_c^d (x_R - x_L)\,dy$
2. What is the formula for vertical strip? ANSWER: $A=\int_a^b (y_U - y_L)\,dx$
3. If the limits are not given we can consider strip. ANSWER: *TRUE*
4. The horizontal strip has x dependent variable and y constant. ANSWER: *TRUE*
5. The vertical strip has y dependent variable and x constant. ANSWER: *TRUE*

PROBLEM-SOLVING

Find the area of the following:

6. Given the area in the first quadrant bounded by $x^2=6y$, the line $y-2=0$ and the y-axis. What is the volume generated when the area is revolved about the line $y-2=0$. ANSWER: *23.2166*
7. The area bounded by the curves $x^2=y$ and the line $x=4$ and the x-axis in the first quadrant is revolved about the y-axis. What is the volume generated? ANSWER*: 402.1238*
8. Determine the volume of the solid obtained by rotating the region bounded by $x=y^2-6y+12$ and $x=7$ about the y-axis. ANSWER: *361.9115*
9. Determine the volume of the solid by rotating the region bounded by $y=9-x^2$, $x=-3$, $x=3$ and the x-axis about the x-axis. ANSWER: *814.3005*
10. Determine the volume of the solid obtained by rotating the region bounded by $y= 6-x^2$, $x=3$, $x=-3$ and the x axis about the x axis. ANSWER: *193.2*

CYLINDRICAL SHELL METHOD

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. The horizontal and vertical line in a line graph are usually called x-axis and y-axis.

Ans: True

2. A horizontal line must intersects the graph of y=f(x) at least one point.

Ans: False

3. This is called the vertical line test for determining whether a curve is the graph of a function.

Ans: True

4. If the cross sections of the solid are taken parallel to the axis of revolution, then the csm will be used to find the volume of the solid.

Ans: True

5. A solid of revolution is formed by revolving a shape around an axis.

Ans: False

PROBLEM-SOLVING

Find the area of the following:

6. Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1. Ans: $\frac{\pi}{2}$

7. Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$. Ans: $\frac{\pi}{2}$

8. Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$. Ans: $\frac{\pi}{6}$

9. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$. Ans: $\frac{16}{5}\pi$

10. Find the volume of the solid generated by revolving the triangular region bounded by $y = x$, $y = 0$, and $x = 2$ about the line $x = 3$. Ans: $\frac{20}{\pi}$

VARIGNON'S THEOREM PROBLEM-SOLVING

Find the area of the following:

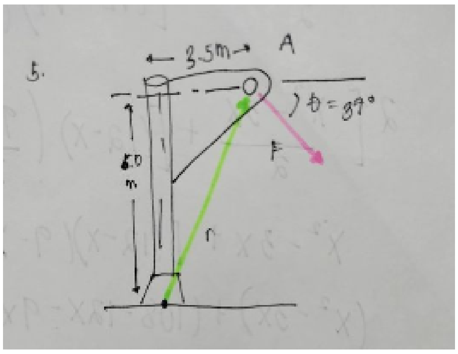
1. Determine the area of $y^2 = x;\ \ y = 3; x = 0\ \ about\ \ the\ \ y - axis.$

2. Prove that $\overline{x} = \dfrac{3a}{5}$ and $\overline{y} = \dfrac{3b}{8}$ for the part of parabolic area bounded by x-axis, line x=a and parabola $y^2 = kx.$

3. Determine the area of $x + 2y = 6, x = 0, y = 0$.

4. Determine the area of $y = \sin x,\ \ y = 0\ \ from\ \ x = 0 - \pi.$

5. Calculate the moment of the force F around the point O shown in the figure if the magnitude of F is 725 N.



TRUE OR FALSE

Indicate whether the following statements are true or false.

6. The location of the centroid is often denoted with a C with the coordinates being $(\overline{x}, \overline{y})$, denoting that they are the average x and y coordinate for the area. Ans. True

7. In the mechanics course sequence, centroids of areas are helpful in a variety of scenarios, such as the study of distributed forces, the bending in beams and torsion in shafts, as well as a preliminary step in calculating moments of inertia. Ans. True

8. First moment integral and method of composite parts are used to determine centroid location.

Ans. True

9. The geometric center of a region is what is known as its centroid. Ans. True

10. According to the centroid theorem, the triangle's centroid is located 2/3 of the way between the vertex and the side midpoints. Ans. True

CENTROID OF SOLID REVOLUTION

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. When finding the centroid of a two-dimensional shape, we will be looking for both x and a y coordinate. Ans: True
2. We use the first moment integral to determine the centroid location. Ans: True
3. The Z_y in Varignon's is the distance of centroid of dV from x-axis. Ans: True
4. The formula of the centroid is $A\bar{x} = \int x_{cen}\, dA$. Ans: True
5. The formula in finding the centroid is $A\bar{y} = \int y_{cen}\, dy$. Ans: False, ($A\bar{y} = \int y_{cen}\, dA$)

PROBLEM-SOLVING

Find the area of the following:

6. Determine the centroid of a solid generated by revolving the region bound by $y = x^4 + 3$, $x = 0$, $y = 0$, and $x = 3$ about the x-axis. Ans: $(\bar{x}, \bar{y}) = \left(\frac{309}{116}, 0\right)$
7. Determine the centroid of a solid generated by revolving the region bound by $y = 5 - x^2$, $x = 0$ and $y = 0$ about the y-axis. Ans: $(\bar{x}, \bar{y}) = \left(0, \frac{5}{3}\right)$
8. Find the centroid of the area bounded by the parabola $y^2 = 4x$, the x-axis and the line $x = 4$. Ans: $(\bar{x}, \bar{y}) = (\frac{12}{5}, \frac{3}{2})$
9. Find the centroid of the first quadrant region bounded by the curve $y^2 = 3x$, the y=axis and the lines $y = 3$ and $y = 6$. Ans: $(\bar{x}, \bar{y}) = (\frac{279}{70}, \frac{135}{28})$
10. Find the centroid of the region bounded by the curves $y = x^2$ and $y = 12x + 1$.

Ans: $(\bar{x}, \bar{y}) = \left(\frac{555}{32}, \frac{2253}{20}\right)$

MOMENT OF INERTIA

TRUE OR FALSE

Indicate whether the following statements are true or false.

1. The strip must be parallel to the axis of rotation. Ans: True

2. Moment of Inertia is known as the capacity of a cross section to not resist bending. Ans: False, (to resist bending.

3. The moment of inertia must be specified with respect to a chosen axis of rotation. Ans: True

4. The formula of the moment of inertia in x-axis for rectangle is $I_x = \frac{bd^3}{12}$. Ans: True

5. The formula of the moment of inertia in x-axis for circle is $I_x = \frac{\pi R^4}{2}$. Ans: False, $(I_x = \frac{\pi R^4}{4})$

PROBLEM-SOLVING

Find the area of the following:

6. What is the moment of inertia of the area in the first quadrant bounded by the curves $y^2 = 8x$, x-axis and the line $x = 2$ with respect to the x-axis? Ans: $\frac{256}{15}$ or 17.06667

7. Find the moment of inertia of the area bounded by $y = 0, f(x), x = 0$ and $x = 15, f(x) = x^3 + 6x^2 - 4x + 5$ with respect to the y-axis? Ans: $\frac{5529375}{2}$ or 2.8×10^6

8. Find the moment of inertia of the area bounded by the curve $x^2 = 3y$, the line $y = 3$ and the y-axis on the first quadrant with respect to the x-axis. Ans: $\frac{54}{5}$ or 10.8

9. Determine the moment of inertia about the x-axis and the y-axis of the area bounded by $y = 0.75x^2$, the x-axis, and $x = 4$. Ans: $I_x = \frac{2304}{7}$, $I_y = \frac{768}{5}$

10. Determine the moment of inertia about the x-axis and the y-axis of the area bounded by $x^2 = 6y$ and the $y = x$. Ans: $I_x = \frac{324}{7}$, $I_y = \frac{324}{5}$

SOLUTION

POWER FORMULA

6. $\int 7x^6 dx$

$= \frac{7x^{6+1}}{6+1} + C$

$= x^7 + C$

7. $\int(3x^2 - \sqrt{9x} + 7dx$

$= 3\int x^2 dx - \sqrt{9}\int\sqrt{x}dx + 7\int dx$

$= 3\frac{x^3}{3} - \sqrt{9}\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 7x + C$

$= x^3 - \sqrt{9}\bullet\frac{2}{3}\sqrt{x^3} + 7x + C$

$= x^3 - \frac{2}{3}x\sqrt{9x} + 7x + C$

8. $\int\left(7x^6 - 8x^7\right)dx$

$= 7\int x^6 dx - 8\int x^7 dx$

$= 7\bullet\frac{x^{6+1}}{6+1} - 8\bullet\frac{x^{7+1}}{7+1} + C$

$= \frac{7x^7}{7} - \frac{8x^8}{8} + C$

$= x^7 - x^6 + C$

9. $\int(8x + 4)(2x + 5)dx$

$= \int 16x^2 + 40x + 8x + 20)dx$

$= \int\left(16x^2 + 48x + 20\right)dx$

$= 16\int x^2 dx + 48\int xdx + 20\int dx$

$= 16\frac{x^{2+1}}{2+1} + 48\frac{x^{1+1}}{1+1} + 20x + C$

$= 16\frac{x^3}{3} + 48\frac{x^2}{2} + 20x + C$

$= 16\frac{x^3}{3} + 24x^2 + 20x + C$

10. $\int\sqrt[4]{x^7}$

$= \int x^{\frac{7}{4}} dx$

$= x^{\frac{11}{4}}\bullet\frac{4}{11} + C$

$= \frac{4}{11}\sqrt[4]{x^{11}} + C$

EXPONENTIAL FUNCTION

6. $\int(2\text{-}3e^x)dx$

$=\int(2 - 3e^x)dx = \int 2dx - \int 3e^x dx$

$=\int 2dx - 3\int e^x dx$

$=2x - 3e^x + C$

7. $\int e^{x}(1+2e^{x})^{4}dx$

$u = 1 + 2e^{x}$ $du = 2e^{x}dx$

or $(1/2)\, du = e^{x}dx$

$=\int e^{x}(1+2e^{x})^{4}dx = \int (1+2e^{x})^{4}e^{x}dx$

$=\int u^{4}(1/2)du$

$=(1/2)\int u^{4}du$

$=(1/2)\,\frac{u^{5}}{5} + C$

$=(1/10)\, u^{5} + C$

$=(1/10)\,(1 + 2e^{x})^{5} + C$

8. $\int (e^{4x} - e^{-4x})^{2}dx$

$(A - B)^{2} = A^{2} - 2AB + B^{2}$

$=\int (e^{4x} - e^{-4x})^{2}dx = \int (e^{4x}e^{4x} - 2e^{4x}e^{-4x} + e^{-4x}e^{-4x})dx$

$=\int (e^{4x + 4x} - 2e^{4x - 4x} + e^{-4x - 4x})dx$

$=\int (e^{8x} - 2e^{0} + e^{-8x})dx$

$=\int (e^{8x} - 2(1) + e^{-8x})dx$

$=\int e^{8x}dx - \int 2dx + \int e^{-8x}dx$

$=\frac{e^{8x}}{8} - 2x + \frac{e^{-8x}}{-8} + C$

$=\frac{e^{8x}}{8} - 2x - \frac{e^{-8x}}{8} + C$

9. $\int e^{x}(1 - e^{x})(1 + e^{x})^{10}dx$

$u = 1 + e^{x} \qquad du = e^{x}dx$

$=\int e^{x}(1 - e^{x})(1 + e^{x})^{10}dx = \int(1 - e^{x})(1 + e^{x})^{10}e^{x}dx$

$=\int(1 - (u - 1))(u)^{10}du$

$=\int(1 - u + 1)u^{10}du$

$=\int(2 - u)u^{10}du$

$=\int(2u^{10} - uu^{10})du$

$=\int(2u^{10} - u^{11})du$

$=2\,\frac{u^{11}}{11} - \frac{u^{12}}{12} + C$

$(2/11)(1 + e^{x})^{11} - (1/12)(1 + e^{x})^{12} + C$

10. $\int e^{5x}(\frac{e^{2x}}{7} + \frac{3}{e^{3x}})dx$

$=\int e^{5x}(\frac{e^{2x}}{7} + \frac{3}{e^{3x}})dx = \int(\frac{e^{5x}e^{2x}}{7} + \frac{3e^{5x}}{e^{3x}})dx$

$=\int(\frac{e^{5x+2x}}{7} + 3e^{5x-3x})dx$

$=\int(\frac{e^{7x}}{7} + 3e^{2x})dx$

$=(1/7)\int e^{7x}dx + 3\int e^{2x}dx$

$=(1/7)\int \frac{e^{7x}}{7} + 3\,\frac{e^{2x}}{2} + C$

$=(1/49)\,e^{7x} + (3/2)\,e^{2x} + C$

SPECIAL COMBINATION

6. $\int (\csc 3y + \cot 3y)^2 \, dy = \int \left(\csc^2 3y + 2 \csc 3y \cot 3y + \cot^2 3y\right) dy$

$= \int \left(\csc^2 3y + 2\csc 3y \cot 3y + \left(\csc^2 3y - 1\right)\right) dy$

$= \int \left(2\csc^2 3y + 2\csc 3y \cot 3y - 1\right) dy$

$= \int \left(2\csc^2 3y + 2\csc 3y \cot 3y\right) dy - \int 1 \, dy$

$= \int \left(2\csc^2 3y + 2\csc 3y \cot 3y\right) dy - y$

Let $u = 3y$

$du = 3dy$

$\left(\frac{1}{3}\right) du = dy$

$= \int \left(2\csc^2 u + 2\csc u \cot u\right)\left(\frac{1}{3}\right) du - y$

$= (2)\left(\frac{1}{3}\right)\int \left(\csc^2 u + \csc u \cot u\right) du - y$

$= \left(\frac{2}{3}\right)\left(\int \csc^2 u \, du + \int \csc u \cot u \, du\right) - y$

$= \left(\frac{2}{3}\right)\left((-\cot u) + (-\csc u)\right) - y$

$= -\frac{2}{3}\cot u - \frac{2}{3}\csc u - y + C$

$= -\frac{2}{3}\cot 3y - \frac{2}{3}\csc 3y - y + C$

7. $\int \tan^2 y \sec y \, dy$

$= \int \left(\sec^2 y - 1\right) \sec y \, dy$

$= \int \sec^3 y \, dy - \int \sec y \, dy$

$= \left[\frac{\sec y \tan y}{2} + \frac{1}{2}\int \sec y \, dy\right] - \int \sec y \, dy$

$= \frac{\sec y \tan y}{2} - \frac{1}{2}\int \sec y \, dy$

$= \frac{\sec y \tan y}{2} - \frac{1}{2} \ln|\sec y + \tan y| + C$

8. $\int 5 \sec 4\theta \tan 4\theta \, d\theta$

Let $u = 4\theta$

$du = 4d\theta$

$\left(\frac{1}{4}\right) du = d\theta$

$= 5 \int \sec u \tan u \left(\frac{1}{4}\right) du$

$= \frac{5}{4} \int \sec u \tan u \, du$

$= \frac{5}{4}\sec u + C$

$= \frac{5}{4}\sec 4\theta + C$

9. $\int \sec^4 3x \tan^3 3x \, dx$

$= \int \sec^2 3x \sec^2 3x \tan^3 3x \, dx$

$= \int \sec^2 3x \left(\tan^2 3x + 1\right) \tan^3 3x \, dx$

$= \int \sec^2 3x \tan^5 3x \, dx + \int \sec^2 3x \tan^3 3x$

$= \frac{1}{18}\tan^6 3x + \frac{1}{12}\tan^4 3x + C$

10. $\int \cot^4 y \, dy$

$= -\frac{\cot^3 y}{3} - \int \cot^2 y \, dy$

$= -\frac{\cot^3 y}{3} - \left(-\cot y - \int 1 dy\right)$

$= -\frac{\cot^3 y}{3} + \cot y + y + C$

TRIGONOMETRIC SUBSTITUTION

6. Evaluate $\int 6sec\, 3x\, tan\, x\, 3x\, dx$

$= 6\int sec\, 3x\, tan\, x\, 3x\, dx$

Let u= 3x   du=3   du/3=dx

$= 6\int \sec sec\, u\ \ \tan tan\, u \frac{du}{3}$

$= 6\bullet\frac{1}{3}\int \sec sec\, u \tan tan\, u\, du$

$= 2\int \sec sec\, u\ \ \tan tan\, u\, du$

$= 2\bullet \sec sec\, u\ + C$

$= 2sec3x + C$

7. Evaluate $\int cot^3 ax\, csc^2 ax\, dx$

Let u= cot ax   du= -csc²ax•a dx   du= -a csc²axdx   -du/a= csc²axdx

$= \int u^3\bullet - \frac{du}{a}$

$= \ - \frac{1}{a}\int u^3 du$

$= \ - \frac{1}{a}\bullet\frac{u^4}{4} + C$

$= \ - \frac{1}{4a}u^4 + C$

$=- \frac{1}{4a}\left(cot^4 ax\right) + C$

8. Evaluate $\int 7cos^2 9xdx$

$= 7\int cos^2 9xdx$

$\cos^2\Theta = \frac{1}{2}(1 + cos2\Theta)$

$= 7\int \frac{1}{2}(1 + cos2(9x)dx$

$= \frac{7}{2}\int(1 + cos18x)dx$

$= \frac{7}{2}\int dx + \frac{7}{2}\int cos\ 18x\, dx$

Let u= 18x   du= 18dx   du/18=dx

$= \frac{7}{2} \int cosu \bullet \frac{du}{18}$

$= \frac{7}{2} \bullet \frac{1}{18} \int \cos cos\, u\, du$

$= \frac{7}{36} \int cos\, u\, du$

$= \frac{7}{36} sinu$

$= \frac{7}{36} sin18x$

$= \frac{7}{2} x + \frac{7}{36} sinx + C$

9. Evaluate $\int (csc6x + cot6x)^2 dx$

$= \int \left(csc^2 6x + 2csc6xcot6x + cot^2 6x \right) dx$

$= \int \left(csc^2 6x + 2csc6xcot6x + csc^2 6x - 1 \right) dx$

$= \int 2csc^2 6x + 2csc6xcot6x - 1) dx$

$= 2 \int csc^2 6xdx + 2 \int csc6xcot6xdx - \int dx$

Let u= 6x   du=6dx   du/6= dx

$= 2 \int csc^2 u \bullet \frac{du}{6} + 2 \int cscu\, cotu \bullet \frac{du}{6}$

$= \frac{2}{6} \int csc^2 u\, du + \frac{2}{6} \int cscu \cot cot\, u\, du$

$= \frac{2}{6} (- cotu) + \frac{2}{6} (- cscu)$

$= - \frac{2}{6} cot6x - \frac{2}{6} csc6x - x + C$

10 Evaluate $\int \tan tan\, 7x\, dx$

Let u= 7x   du= 7dx   du/7=dx

$= \int \tan tan\, u \bullet \frac{du}{7}$

$= \frac{1}{7} \int \tan tan\, u\, du$

$= \frac{1}{7} \bullet (- \ln ln\, |cosu|) + C$

$= \frac{1}{7} \ln ln\, |secu| + C$

$= - \frac{1}{7} ln|cos(7x)| + C$

HYPERBOLIC FUNCTION

6. $\int \frac{\cosh x}{\cosh^2 x}$

$= \int \frac{1}{\cosh x} dx$

$= \int \cosh x \cdot \frac{1}{\sinh^2 x+1} dx$

$u = \sinh x \qquad du = \cosh x$

$dx = \frac{1}{\cosh x} du$

$= \int \frac{1}{u^2+1} du$

$= \arctan(\sinh(x)) + C$

7. $\int_0^{\ln 4} 4e^x \sinh x \, dx$

$= \ (e^{2x} - 2x)|_0^{\ln 4}$

$= \left[e^{2(\ln 4)} - 2(\ln 4)\right] - \left[e^{2(0)} - 2(0)\right]$

$= (e^{\ln 4^2} - \ln 4^2) - (e^0 - 0)$

$= e^{\ln 16} - \ln 16 - 1$

$= 16 - \ln 16 - 1$

$= 15 - 2\ln 4$ or 12.227

8. $\int e^{3x} \sinh x \, dx$

$\int (\frac{e^{2x}}{4} - \frac{1}{4}) \cdot 2e^{2x} dx$

$u = e^{2x} \qquad du = 2e^{2x}$

$dx = \frac{e^{-2x}}{2} dx$

$= \frac{1}{4}\int (u-1) du$

$\int (u-1) du$

$= \int u \, du - \int 1 du$

$= \frac{u^2}{2} - u$

$= \frac{1}{4}\int (u-1) du$

$= \frac{u^2}{8} - \frac{u}{4}$

$= \frac{e^{4x}}{8} - \frac{e^{2x}}{4} + C$

$= \frac{e^{4x} - 2e^{2x}}{8} + C$

9. $\int \frac{\cosh x}{6+5\sinh x} dx$

$u = 6 + 5\sinh x\,, \qquad du = 5\cosh x \, dx,$

$\cosh x \, dx = \frac{du}{5}$

$\int \frac{\cosh x}{6+5\sinh x} dx$

$= \int \frac{\frac{du}{5}}{u}$

$= \frac{1}{5}\int \frac{du}{u}$

$= \frac{1}{5}\ln u + C$

$= \frac{1}{5}\ln(6 + 5\sinh x) + C$

10. $\int 6e^x \sinh x \, dx$

$= \int 6e^x \cdot (\frac{e^x - e^{-x}}{2}) \, dx$

$= \int 3(e^{2x} - e^0) \, dx$

$= 3\int (e^{2x} - 1) \, dx$

$= 3\left(\frac{1}{2}e^{2x} - x\right) + C$

$= \frac{3}{2}e^{2x} - 3x + C$

INVERSE TRIGONOMETRIC FUNCTION

6. Find the values of sin(cos $^{-1}$ 5/6)

$Let, \frac{5}{6} = \theta$

$cos\theta = 5/6$

sin θ= $\sqrt{(1 - cos^2\theta)}$

$= \sqrt{1 - (5/6)^2}$

$= \sqrt{1 - \frac{25}{36}}$

$= \sqrt{\frac{11}{36}}$

$= \frac{\sqrt{11}}{6}$

7. Find the values of cos(tan $^{-1}$ 2/3)

$Let, \frac{2}{3} = \theta$

$tan\theta = \frac{2}{3}$

$sec^2\theta$-tan^2 θ=1

sec θ=$\sqrt{(1 + tan^2\theta)}$

$= \sqrt{\left(1 + \left(\frac{2}{3}\right)^2\right)}$

$= \sqrt{\left(1 + \frac{4}{9}\right)}$

$= \sqrt{\frac{13}{9}}$

8. Evaluate $\int \frac{xdx}{\sqrt{16-81x^4}}$

$u^2=81x^4$ $u=9x^2$ du/18= 18xdx/18 du/18=xdx $a^2=16$ a=4

$\int \frac{du}{18\left(\sqrt{a^2-u^2}\right)}$

$= \frac{1}{18}\int \frac{du}{\sqrt{a^2-u^2}}$

$= \frac{1}{18}\left(\frac{u}{a}\right) + C$

$= \frac{1}{8}\left(\frac{9x^2}{4}\right) + C$

9. Evaluate $\int \frac{dx}{144\sqrt{x}+x\sqrt{x}}$

$= \int \frac{dx}{\sqrt{x}(144+x)}$

$u^2= x$ u=√x 2[du=dx/2√x] 2du=dx/x $a^2=144$ a=12

$= \int \frac{dx}{\sqrt{x}(12^2+(\sqrt{x})^2}$

$= \int \frac{2du}{a^2+u^2}$ $= 2\int \frac{du}{a^2+u^2}$ $= 2[\frac{1}{a}(\frac{u}{a})] + C$

$= \frac{2}{12}\frac{\sqrt{x}}{12} + C$

10.Evaluate $\int \frac{10}{x\sqrt{8x^2-1}}$

$u^2=8x^2$ $u=4x$ $du=2dx$ $du/2=dx$ $a^2=1$ $a=1$ $x=u/2$

$$= 10\int \frac{1}{\frac{u}{2}\left(u^2-a^2\right)} \frac{du}{2}$$

$$= 10 \frac{du}{u\sqrt{u^2-a^2}} \quad = 10 \bullet \frac{1}{a}\left(\frac{u}{a}\right) + C$$

$$= 10(4x) + C$$

INTEGRALS OF TERMS INVOLVING U2+ A2 AND A2- U2

1. Evaluate $\int \frac{3}{x\sqrt{x^2-9}} dx$

$$= 3\int \frac{1}{x\sqrt{x^2-9}} dx$$

$$= 3\int \frac{1}{x\sqrt{x^2-3^2}} dx$$

$$= 3 \ \frac{1}{3} \ asec(\frac{|x|}{3})$$

$$= asec(\frac{|x|}{3}) + C$$

2. Evaluate $\int \frac{e^x}{\sqrt{7-e^{2x}}} dx$

$$= \int \frac{1}{\sqrt{7-u^2}} du$$

$$= arcsin(\frac{u}{\sqrt{7}})$$

$$= arcsin(\frac{e^x}{\sqrt{7}})$$

$$= arcsin(\frac{\sqrt{7}\,e^x}{\sqrt{7}}) + C$$

3. Evaluate $\int \frac{1}{\sqrt{x^2-a^2}} dx$

$$= In\ (|x + \sqrt{x^2 - a^2|})$$

$$=In\ (|x + \sqrt{x^2 - a^2|}) + C$$

4. Evaluate $\int \frac{\sin x}{\sqrt{2-\cos^2 x}} dx$

$$= \int - \frac{1}{\sqrt{2-u^2}} du$$

$$= - \int \frac{1}{\sqrt{2-u^2}} du$$

$$= - \ arcsin(\frac{u}{\sqrt{2}})$$

$$= - \ arcsin(\frac{\cos(x)}{\sqrt{2}})$$

$$= - \ arcsin(\frac{\sqrt{2}\cos(x)}{2}) + C$$

5. Evaluate $\int \frac{1}{\sqrt{1-4x^2}} dx$

$$= \int \frac{1}{\sqrt{4(\frac{1}{4}-x^2)}} dx$$

$$= \int \frac{1}{\sqrt{4}\sqrt{\frac{1}{4}-x^2}} dx$$

$$= \frac{1}{\sqrt{4}} \int \frac{1}{\sqrt{\frac{1}{4}-x^2}} dx$$

$$= \frac{1}{2} \ arcsin\ (\frac{x}{\frac{1}{2}})$$

$$= \frac{arcsin\,(2x)}{2} + C$$

INTEGRATION OF POWERS & PRODUCTS OF SIN & COS

1. $\int x^2 \cos x \ dx$

Let $u = x^2$

$du = 2x\, dx$

$v = \sin x$

$dv = \cos x \ dx$

$=x^2 \sin x - \int 2x \sin x \ dx$

$=x^2 \sin x - 2 \int x \sin x \ dx$

Let $u = x$

$du = dx$

$v = -\cos x$

$dv = \sin x \ dx$

$=x^2 \sin x - 2\left[-x \cos x - \int (-\cos x) dx\right]$

$=x^2 \sin x - 2\left[-x \cos x + \int \cos x \, dx\right]$

$=x^2 \sin x - 2[-x \cos x + \sin x]$

$=x^2 \sin x + 2x \cos x - 2 \sin x + C$

2. $\int \sin 3\,\theta \cos 4\,\theta \, d\theta$

Let $u = \sin 3\,\theta$

$du = 3 \cos 3\,\theta \, d\theta$

$v = \left(\frac{1}{4}\right) \sin 4\,\theta$

$dv = \cos 4\,\theta \, d\theta$

$=\sin 3\,\theta \left(\frac{1}{4}\right) \sin 4\,\theta - \int \left(\frac{1}{4}\right) \sin 4\,\theta (3) \cos 3\,\theta \, d\theta$

$=\left(\frac{1}{4}\right) \sin 3\,\theta \sin 4\,\theta - \left(\frac{3}{4}\right) \int \cos 3\,\theta \sin 4\,\theta \, d\theta$

Let $u = \cos 3\,\theta$

$du = -3 \sin 3\,\theta \, d\theta$

$v = \left(-\frac{1}{4}\right) \cos 4\,\theta$

$dv = \sin 4\,\theta \, d\theta$

$=\left(\frac{1}{4}\right) \sin 3\,\theta \sin 4\,\theta - \left(\frac{3}{4}\right) \left[\cos 3\,\theta \left(-\frac{1}{4}\right) \cos 4\,\theta - \int \left(-\frac{1}{4}\right) \cos 4\,\theta (-3) \sin 3\,\theta d\theta\right]$

$=\left(\frac{1}{4}\right)\sin 3\theta \sin 4\theta - \left(\frac{3}{4}\right)\left[\left(-\frac{1}{4}\right)\cos 3\theta \cos 4\theta - \left(\frac{3}{4}\right) \int \sin 3\theta \cos 4\theta \, d\theta\right]$

$=\left(\frac{1}{4}\right)\sin 3\theta \sin 4\theta + \left(\frac{3}{16}\right)\cos 3\theta \cos 4\theta + \left(\frac{9}{16}\right) \int \sin 3\theta \cos 4\theta \, d\theta$

$$\left(\frac{7}{16}\right) \int \sin 3\theta \cos 4\theta \, d\theta = \left(\frac{1}{4}\right)\sin 3\theta \sin 4\theta + \left(\frac{3}{16}\right)\cos 3\theta \cos 4\theta$$

$$\int \sin 3\theta \cos 4\theta \, d\theta = \left(\frac{16}{7}\right)\left(\left(\frac{1}{4}\right)\sin 3\theta \sin 4\theta + \left(\frac{3}{16}\right)\cos 3\theta \cos 4\theta\right)$$

$=\left(\frac{4}{7}\right)\sin 3\theta \sin 4\theta + \left(\frac{3}{7}\right)\cos 3\theta \cos 4\theta + C$

3. $\int \sin^3 y \cos^2 y \, dy$

$= \int \sin^2 y \cos^2 y \cdot \sin y \, dy$

$= \int (1-\cos^2 y)\cos^2 y \cdot \sin y \, dy$

Let

$du = -\sin y$

$= \int (1-u^2)u^2 \, du$

$= \int u^2 + u^4 du$

$= \frac{u^3}{3} + \frac{u^5}{5}$

$= \frac{\cos^3 y}{3} + \frac{\cos^5 y}{5} + C$

4. $\int x \sin 3x \, dx$

Let $u = x$

$du = dx$

$v = \left(-\frac{1}{3}\right)\cos 3x$

$dv = \sin 3x \, dx$

$=x\left(-\frac{1}{3}\right)\cos 3x - \int \left(-\frac{1}{3}\right)\cos 3x \, dx$

$=-\frac{x}{3}\cos 3x + \left(\frac{1}{3}\right) \int \cos 3x \, dx$

$=-\frac{x}{3}\cos 3x + \left(\frac{1}{3}\right) \frac{\sin 3x}{3} + C$

$=-\frac{x}{3}\cos 3x + \left(\frac{1}{9}\right) \sin 3x + C$

5. $\int (\sin y + \cos y)^2 \, dy$

$= \int (\sin^2 y + 2\sin y \cos y + \cos^2 y) \, dy$

$= \int \left((\sin^2 y + \cos^2 y) + 2\sin y \cos y\right) dy$

Let $u = \sin y$

$du = \cos y \, dy$

$$x + 2\int \sin y \cos y \, dy = x + 2\int u \, du$$

$x + 2\,\frac{u^2}{2}$

$x + u^2$

$x + \sin^2 y + C$

INTEGRATION BY PARTS

6. $\int_0^{\pi} a^2 \cos(6a)\, da$

$u = a^2 \qquad dv = \cos(6a)\, da$

$du = 2a\, da \qquad v = \frac{1}{6}\sin(6a)$

$\int udv = uv - \int vdu$

$= \frac{1}{6}a^2\sin(6a) - \frac{1}{3}\int a\sin(6a)da$

$u = a \qquad dv = \sin(6a)da$

$du = da \qquad v = -\frac{1}{6}\cos(6a)$

$\int a^2 \sin(6a)\, da$

$= \frac{1}{6}a^2\sin(6a) - \frac{1}{3}\left[-\frac{1}{6}a\cos(6a) + \frac{1}{6}\int\cos(6a)da\right]$

$= \frac{1}{6}a^2\sin(6a) - \frac{1}{3}\left[-\frac{1}{6}a\cos(6a) + \frac{1}{36}\sin(6a)\right]$

$= \frac{1}{6}a^2\sin(6a) + \frac{1}{18}\, a\cos(6\mathrm{a}) - \frac{1}{96}\sin(6a)$

$$= \frac{1}{6}a^2\sin(6a) + \frac{1}{18}\, a\cos(6\mathrm{a}) - \frac{1}{96}\sin(6a)\Big|_0^{\pi}$$

$= \left[\frac{1}{6}(\pi)^2\sin(6(\pi)) + \frac{1}{18}(\pi)\cos(6(\pi)) - \frac{1}{96}\sin(\pi)\right] - \left[\frac{1}{6}(0)^2\sin(6(0)) + \frac{1}{18}(0)\cos(6(0)) - \frac{1}{96}\sin(0)\right]$

$= \frac{\pi}{18}$

7. $\int_0^5 (3+5x)e^{\frac{1}{3}x}\, dx$

$u = 3+5x \qquad dv = e^{\frac{1}{3}x}$

$du = 5dx \qquad v = 3e^{\frac{1}{3}x}$

$\int(3+5x)e^{\frac{1}{3}x}dx$

$= (3+5x)\left(3e^{\frac{1}{3}x}\right) - \int 5\left(3e^{\frac{1}{3}x}\right)dx$

$= 3e^{\frac{1}{3}x}(3+5x) - 15\int e^{\frac{1}{3}x}\, dx$

$= 3e^{\frac{1}{3}x}(3+5x) - 45e^{\frac{1}{3}x}$

$= 9e^{\frac{1}{3}x} + 15xe^{\frac{1}{3}x} - 45e^{\frac{1}{3}x}$

$= 15xe^{\frac{1}{3}x} - 36e^{\frac{1}{3}x}\Big|_0^5$

$= \left[15(5)e^{\frac{1}{3}(5)} - 36e^{\frac{1}{3}(5)}\right] - \left[15(5)e^{\frac{1}{3}(0)} - 36e^{\frac{1}{3}(0)}\right]$

$= 242.4851$

8. $\int x^4 \ln x\, dx$

$u = \ln x \qquad v = \frac{x^5}{5}$

$du = \frac{1}{x}dx \qquad dv = x^4dx$

$\int udv = uv - \int vdu$

$= \ln x\left(\frac{x^5}{5}\right) - \int\frac{x^5}{5}\cdot\frac{1}{x}dx$

$= \frac{1}{5}x^5\ln x - \frac{1}{5}\int x^4dx$

$= \frac{1}{5}x^5\ln x - \frac{1}{5}\left(\frac{x^{4+1}}{4+1}\right)$

$= \frac{1}{5}x^5\ln x - \frac{1}{5}\left(\frac{x^5}{5}\right)$

$= \frac{1}{5}x^5\ln x - \frac{1}{25}x^5 + C$

$= \frac{x^5(5\ln(x)-1)}{25} + C$

9. $\int e^{6x}\cos(e^{2x})dx$

$= \int e^{4x}e^{2x}\cos(e^{2x})dx$

$u = e^{4x} \qquad \mathrm{d}v = e^{2x}\cos(e^{2x})dx$

$du = 4e^{4x}dx \qquad dv = \frac{1}{2}\sin(e^{2x})$

$\int udv = uv - \int vdu$

$= \frac{1}{2}e^{4x}\sin(e^{2x}) - 2\int e^{4x}\sin(e^{2x})dx \qquad u = e^{2x},\ du = 2e^{2x}dx$

$= \frac{1}{2}e^{4x}\sin(e^{2x}) - 2\int e^{2x}e^{2x}\sin(e^{2x})dx \qquad dv = e^{2x}\sin(e^{2x})dx,$

$= \frac{1}{2}e^{4x}\sin(e^{2x}) - 2[-\frac{1}{2}e^{2x}\cos(e^{2x}) + \int e^{2x}\cos(e^{2x})dx \qquad v = -\frac{1}{2}\cos(e^{2x})$

$= \frac{1}{2}e^{4x}\sin(e^{2x}) + e^{2x}\cos(e^{2x}) - 2\int e^{2x}\cos(e^{2x})dx$

$= \frac{1}{2}e^{4x}\sin(e^{2x}) + e^{2x}\cos(e^{2x}) - 2(\frac{1}{2}\sin(e^{2x}) + C$

$= \frac{1}{2}e^{4x}\sin(e^{2x}) + e^{2x}\cos(e^{2x}) - \sin(e^{2x}) + C$

$= \frac{(e^{4x}-2)\sin(e^{2x})}{2} + e^{2x}\cos(e^{2x}) + C$

10. $\int e^{4x} \cos(2x)dx$

$u = \cos(2x) \qquad dv = e^{4x}\, dx$

$du = -2\sin(2x)\, dx \qquad v = \frac{e^{4x}}{4}$

$\int udv = uv - \int vdu$

$I = \frac{1}{4}e^{4x}\cos(2x) + \int \frac{2}{4}e^{4x}\sin(2x)\text{dx} \qquad u = \sin(2x), \quad du = 2\cos(2x)\, dx,$

$dv = \frac{2}{4}e^{4x}\, dx, \; v = \frac{2}{16}e^{4x}$

$I = \frac{1}{4}e^{4x}\cos(2x) + \frac{2}{16}e^{4x}\sin(2x) - \frac{4}{16}\int e^{4x}\cos(2x)dx \qquad I = \int e^{4x}\cos(2x)dx$

$\frac{16}{16} + \frac{4}{16}I = \frac{1}{4}e^{4x}\cos(2x) + \frac{2}{16}e^{4x}\sin(2x) - \frac{4}{16} + \frac{4}{16}\, I$

$\frac{16}{20}\cdot\frac{20}{16}I = \frac{16}{20}\cdot\frac{1}{4}e^{4x}\cos(2x) + \frac{16}{20}\frac{2}{16}e^{4x}\sin(2x)$

$I = \frac{4}{20}e^{4x}\cos(2x) + \frac{2}{20}e^{4x}\sin(2x) + C$

$= \frac{e^{4x}(\sin(2x) + 2\cos(2x))}{10} + C$

INTEGRATION BY TRIGONOMETRIC SUBSTITUTION

6. $\int \frac{\sqrt{16-x^2}}{x^2}dx$

$a^2 = 16, \qquad a = 4$

$x = 4\sin\theta, \qquad dx = 4\cos\theta d\theta$

$= \int \frac{\sqrt{16-x^2}}{x^2}$

$= \int \frac{\sqrt{16-(4\sin\theta)^2}}{(4\sin\theta)^2}\cdot 4\cos\theta\; d\theta$

$= \int \frac{\sqrt{16-16\sin^2\theta}}{16\sin^2\theta}\cdot 4\cos\theta\; d\theta$

$= \int \frac{\sqrt{16(1-\sin^2\theta)}}{4\sin^2\theta}\cdot \cos\theta\; d\theta$

$= \int \frac{4\sqrt{\cos^2\theta}}{4\sin^2\theta}\cdot \cos\theta\; d\theta$

$= \int \frac{\cos\theta}{\sin^2\theta}\cdot \cos\theta\; d\theta$

$= \int \frac{\cos^2\theta}{\sin^2\theta} \qquad x = 4\sin\theta,$

7. $\int \sqrt{1-6y^2}\, dy$

$y = \frac{1}{\sqrt{6}}\sin\theta$

$\sqrt{1-6y^2}$

$= \sqrt{1-\sin^2\theta}$

$= \sqrt{1-\cos^2\theta}$

$= |\cos\theta|$

$dy = \frac{1}{\sqrt{6}}\cos\theta\; d\theta$

$\int \sqrt{1-6y^2}$

$= \int \cos\theta\; (\frac{1}{\sqrt{6}}\cos\theta)\; d\theta$

$= \frac{1}{\sqrt{6}}\int \cos^2\theta\;\; d\theta$

$= \int \frac{1-\sin^2\theta}{\sin^2\theta} d\theta$

$= \int [\frac{1}{\sin^2\theta} - \frac{\sin^2\theta}{\sin^2\theta}]$

$= \int [\csc^2\theta - 1]\, d\theta$

$= -\cot\theta - \theta + C$

$= -\frac{\sqrt{16-x^2}}{x} - \sin^{-1}\left(\frac{x}{4}\right) + C$

$\frac{x}{4} = \sin\theta$

$\sin^{-1}\frac{x}{4} = \sin^{-1}(\sin\theta)$

$\theta = \sin^{-1}\left(\frac{x}{4}\right)$

$\tan\theta = \frac{x}{\sqrt{16-x^2}}$

$\cot\theta = \frac{1}{\tan\theta}$

$= \frac{1}{\sqrt{6}}\int \frac{1}{2}[1+\cos(2\theta)]\, d\theta$

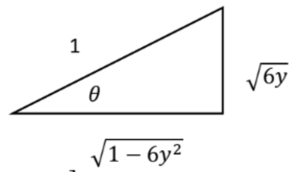
$= \frac{1}{2\sqrt{6}}\left[\theta + \frac{1}{2}\sin(2\theta)\right] + C$

$= \frac{1}{2\sqrt{6}}[\theta + \sin(\theta)\cos(\theta)] + C$

$\sin\theta = \sqrt{6} \quad \to \quad \theta = \sin^{-1}(\sqrt{6}y)$

$\sin\theta = \frac{opp}{hyp} = \frac{\sqrt{6y}}{1}$

$\cos\theta = \sqrt{1-6y^2}$



$= \frac{1}{2\sqrt{6}}\left[\sin^{-1}(\sqrt{6y}) + \sqrt{6y}\sqrt{1-6y^2}\right] + C$

8. $\int \cos x\sqrt{4+25\sin^2 x}\, dx = \int \sqrt{4+25\,[\sin x]^2}$

$u = \sin x \qquad du = \cos x\, dx$

$dx = \frac{1}{\cos x} du$

$\int \sqrt{25u^2+4}du$

$u = \frac{2\tan v}{5} \qquad v = \arctan\left(\frac{5u}{2}\right)$

$\mathrm{du} = \frac{2\sec^2(v)}{5}$

$= \int \frac{2\sec^2(v)\sqrt{4\tan^2(v)+4}}{5} dv$

$= \frac{4}{5}\int \sec^3(v)dv$

$\int \sec^3(v)dv$

$= \frac{\sec v\tan v}{2} + \frac{1}{2}\int \sec v\, dv$

$\int \sec v\, dv$

$= \ln(\tan v + \sec v)$

$= \frac{\sec v\tan v}{2} + \frac{1}{2}\int \sec v\, dv$

$= \frac{\ln(\tan v+\sec v)}{2} + \frac{\sec v \text{ tuan } v}{2}$

$= \frac{4}{5}\int \sec^3(v)dv$

$= \frac{2\ln(\tan v+\sec v)}{5} + \frac{2\sec v\tan v}{5}$

Undo substitution: $u = \arctan(\frac{5u}{2})$

$\tan(\arctan\frac{5u}{2}) = \frac{5u}{2}$

$\sec(\arctan\frac{5u}{2}) = \sqrt{\frac{25u^2}{4}+1}$

$= \frac{2\ln\left(\sqrt{\frac{25\sin^2(x)}{4}+1}+\frac{5\sin x}{2}\right)}{5} + \sin x\sqrt{\frac{25\sin^2(x)}{4}+1} + C$

$= \frac{2\ln(|\sqrt{25\sin^2 x+4}+5\sin x|)}{5} + \sin x\sqrt{\frac{25\sin^2(x)}{4}+1} + C$

9. $\int_1^3 2x^5\sqrt{4+16x^2}dx$

$4\int x^5\sqrt{4x^2+1}\, dx$

$u = 4x^2+1 \qquad du = 8x \qquad dx = \frac{1}{8x}du$

$= \frac{1}{128}\int (u-1)^2\sqrt{u}du$

$= \frac{1}{128}\int \left(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + \sqrt{u}\right) du$

$= \int u^{\frac{5}{2}}\, du - 2\int u^{\frac{3}{2}}du + \int \sqrt{u}du$

$= \frac{2u^{\frac{7}{2}}}{7} - \frac{4u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3}$

$\frac{1}{128}\int (u-1)^2\sqrt{u}du$

$= \frac{u^{\frac{7}{2}}}{448} - \frac{u^{\frac{5}{2}}}{160} + \frac{u^{\frac{3}{2}}}{192}$

$= \frac{(4x^2+1)^{\frac{7}{2}}}{448} - \frac{(4x^2+1)^{\frac{5}{2}}}{160} + \frac{(4x^2+1)^{\frac{3}{2}}}{192}$

$4\int x^5\sqrt{4x^2+1}\, dx$

$= \frac{(4x^2+1)^{\frac{7}{2}}}{112} - \frac{(4x^2+1)^{\frac{5}{2}}}{40} + \frac{(4x^2+1)^{\frac{3}{2}}}{48}$

$= \frac{(4x^2+1)^{\frac{3}{2}}(30x^4-6x^2+1)}{210} + C$

$= 2(\frac{2377.37^{\frac{3}{2}}}{420} - \frac{5^{\frac{5}{2}}}{84})$

$= 2546.1589$

10. $\int_{-7}^{-5} \frac{3}{x^3\sqrt{x^2-25}} dx$

$= 3\int \frac{1}{x^3\sqrt{x^2-25}} dx$

$u = \sqrt{x^2-25}$ $\qquad du = \frac{x}{\sqrt{x^2-25}}$ $\qquad dx = \frac{\sqrt{x^2-25}}{x}$

$= \int \frac{1}{(u^2+25)^2} du$

Apply reduction formula:

$$\int \frac{1}{(au^2+b)^n} du = \frac{2n-3}{2b(n-1)} \int \frac{1}{(au^2+b)^{n-1}} du + \frac{u}{2b(n-1)(au^2+b)^{n-1}}$$

$a = 1, b = 25, n = 2$

$= \frac{u}{50(u^2+25)} + \frac{1}{50}\int \frac{1}{u^2+25} du$

$$\int \frac{1}{u^2+25} du$$

$v = \frac{u}{5}$ $\qquad dv = \frac{1}{5}$ $\qquad du = 5dv$

$= \int \frac{5}{25v^2+25} dv$

$= \frac{1}{5}\int \frac{1}{v^2+1} dv$

$= \frac{1}{5}\int \frac{1}{v^2+1} dv$

$= \frac{\arctan v}{5}$

$= \frac{u}{50(u^2+25)} + \frac{1}{50}\int \frac{1}{u^2+25} du$

$= \frac{u}{50(u^2+25)} + \frac{\arctan(\frac{u}{5})}{250}$

$= \frac{\arctan(\frac{\sqrt{x^2-25}}{5})}{250} + \frac{\sqrt{x^2-25}}{50x^2}$

$3\int \frac{1}{x^3\sqrt{x^2-25}} dx$

$= \frac{3\arctan(\frac{\sqrt{x^2-25}}{5})}{250} + \frac{3\sqrt{x^2-25}}{50x^2}$

$= 3\left(\frac{49\arcsin\left(\frac{5}{7}\right) - 10\sqrt{6}}{12250} - \frac{\pi}{500}\right)$

$= \frac{294\arcsin\left(\frac{5}{7}\right) - 147\pi - 10\cdot 6^{\frac{3}{2}}}{24500}$

$= -0.015301$

RATIONAL FUNCTION

1. $$\int \frac{4x+3}{(x-2)^2(x+3)}dx$$

$$= \int \frac{9}{25(x-2)} + \frac{11}{5(x-2)^2} - \frac{9}{25(x+3)}dx$$

$$= \frac{9}{25}\ln|x-2| - \frac{11}{5(x-2)} - \frac{9}{25}\ln|x+3|$$

$$= \frac{9}{25}\ln|x-2| - \frac{11}{5(x-2)} - \frac{9}{25}\ln|x+3| + C$$

2. $$\int \frac{4}{(x^2-4)^2}dx$$

$$= 4\cdot \int \frac{1}{32(x+2)} + \frac{1}{16(x+2)^2} - \frac{1}{32(x-2)} + \frac{1}{16(x-2)^2}dx$$

$$= 4\left(\int \frac{1}{32(x+2)}dx + \int \frac{1}{16(x+2)^2}dx - \int \frac{1}{32(x-2)}dx + \int \frac{1}{16(x-2)^2}dx \right)$$

$$= 4\left(\frac{1}{32}\ln|x+2| - \frac{1}{16(x+2)} - \frac{1}{32}\ln|x-2| - \frac{1}{16(x-2)} \right)$$

$$= 4\left(\frac{1}{32}\ln|x+2| - \frac{1}{16(x+2)} - \frac{1}{32}\ln|x-2| - \frac{1}{16(x-2)} \right) + C$$

3. $$\int \frac{3x}{(x+1)^2(x-3)}dx$$

$$= 3\cdot \int \frac{x}{(x+1)^2(x-3)}dx$$

$$= 3\bullet \int -\frac{3}{16(x+1)} + \frac{1}{4(x+1)^2} + \frac{3}{16(x-3)}dx$$

$$= 3\left(-\int \frac{3}{16(x+1)}dx + \int \frac{1}{4(x+1)^2}dx + \int \frac{3}{16(x-3)}dx \right)$$

$$= 3\left(-\frac{3}{16}\ln|x+1| - \frac{1}{4(x+1)} + \frac{3}{16}\ln|x-3| \right)$$

$$= 3\left(-\frac{3}{16}\ln|x+1| - \frac{1}{4(x+1)} + \frac{3}{16}\ln|x-3| \right) + C$$

4. $$\int \frac{2}{(x+2)(x-4)}dx$$

$$= 2\cdot \int \frac{1}{(x+2)(x-4)}dx$$

$$= 2\bullet \int -\frac{1}{6(x+2)} + \frac{1}{6(x-4)}dx$$

$$= 2\left(-\int \frac{1}{6(x+2)}dx + \int \frac{1}{6(x-4)}dx \right)$$

$$= 2\left(-\frac{1}{6}\ln|x+2| + \frac{1}{6}\ln|x-4| \right)$$

$$= 2\left(-\frac{1}{6}\ln|x+2| + \frac{1}{6}\ln|x-4| \right) + C$$

5. $$\int \frac{3}{x^2+5x+6}dx$$

$$= 3\cdot \int \frac{1}{x^2+5x+6}dx$$

$$= 3\cdot \int \frac{1}{x+2} - \frac{1}{x+3}dx$$

$$= 3\left(\int \frac{1}{x+2}dx - \int \frac{1}{x+3}dx \right)$$

$$= 3(\ln|x+2| - \ln|x+3|)$$

$$= 3(\ln|x+2| - \ln|x+3|) + C$$

WALLIS FORMULA

6. $\int_0^{\frac{\pi}{2}} sin^9 v\ cos^2 v\ dv$

$= \frac{1}{k}\frac{[(m-1)(m-3)(m-5)]\xi[(n-1)(n-3)(n-5)\xi]}{(m+n)(m+n-2)(m+n-4)(m+n-6)\xi}\cdot\alpha$

$= \frac{[(9-1)(9-3)(9-5)(9-7)(2-1)]}{(9+2)(11-2)(11-4)(11-6)(11-8)(11-10)}\cdot 1$

$= \frac{(8\cdot6\cdot4\cdot2\ )(1)}{(11\cdot9\cdot7\cdot5\cdot3\cdot1)}\ \cdot 1$

$= \ \frac{384}{10395} \div 3$

$= \frac{128}{3465}$

7. $\int_0^{\frac{\pi}{2}} cos^8 a\ sin^5 a\ da$

$= \frac{1}{k}\frac{[(m-1)(m-3)(m-5)\xi][\xi(n-1)(n-3)(n-5)]}{(m+n)(m+n-2)(m+n-4)(m+n-6)\xi}\cdot\alpha$

$= \frac{[(8-1)(8-3)(8-5)(8-7)(5-1)(5-3)]}{(8+5)(13-2)(13-4)(13-6)(13-8)(13-10)(13-12)}\cdot 1$

$= \frac{(7\cdot5\cdot3\cdot1)(4\cdot2)}{(13\cdot11\cdot9\cdot7\cdot5\cdot3\cdot1)}\cdot 1$

$= \frac{840}{135135} \div 3$

$= \frac{8}{1287}$

8. $\int_0^{\frac{\pi}{10}} \sin^6 5x \cos^4 5x\ dx$

Let;

$u = 5x$ at $x = 0,\ u_1 = 5(0) = 0$

$\mathrm{d}u = 5dx$ at $x = \frac{\pi}{10}, u_2 = 5\left(\frac{\pi}{10}\right) = \frac{\pi}{2}$

$\frac{du}{5} = dx$

$\int_0^{\frac{\pi}{10}} \sin^6 5x \cos^4 5x\ dx$

$= \int_{u_1}^{u_2} \sin^6 u \cos^4 u\ \frac{du}{5}$

$= \ \frac{1}{5}\int_0^{\frac{\pi}{2}} \sin^6 u \cos^4 u\ du$

$= \frac{1}{k}\frac{[(m-1)(m-3)(m-5)]\xi[(n-1)(n-3)(n-5)\xi]}{(m+n)(m+n-2)(m+n-4)(m+n-6)\xi}\cdot\alpha$

$= \ \frac{1}{5}\ \frac{(5\cdot3\cdot1)(3\cdot1)}{(10\cdot8\cdot6\cdot4\cdot2)}\cdot\frac{\pi}{2}$

$= \frac{45\pi}{38400}\ \div \frac{15}{15}$

$= \frac{3\pi}{2560}$

9. $\int_0^a x^2(a^2 - x^2)^{\frac{3}{2}}dx$

$\sin\theta = \frac{x}{a}$

$x = asin\theta;\quad dx = a\cos\theta d\theta$

$x^2 = a^2\sin^2\theta$ at $x = 0, \theta_1 = \sin^{-1}\frac{0}{a} = 0$

$a^2 - x^2 = a^2\sin^2\theta$ at $x = a, \theta_2 = \sin^{-1}\frac{a}{a} = \frac{\pi}{2}$

$= a^2\ (1 - \sin^2\theta)$

$(a^2 - x^2 = a^2\cos^2\theta)^{\frac{3}{2}}$

$(a^2 - x^2)^{\frac{3}{2}} = a^3\cos^3\theta$

$\int_0^a x^2(a^2 - x^2)^{\frac{3}{2}}dx$

$= \int_{\theta_1}^{\theta_2} a^2\sin^2\theta\ (a^3\cos^3\theta)a\ \cos\theta\ d\theta$

$= \ a^6\int_0^{\frac{\pi}{2}}\sin^2\theta\cos^4\theta$

$= \frac{1}{k}\frac{[(m-1)(m-3)(m-5)]\xi[(n-1)(n-3)(n-5)\xi]}{(m+n)(m+n-2)(m+n-4)(m+n-6)\xi}\cdot\alpha$

$= \ \ a^6\,\frac{1\cdot3\cdot1}{6\cdot4\cdot2}\cdot\frac{\pi}{2}$

$= \frac{\pi a^6}{32}$

10. $\int_0^{\frac{\pi}{6}} \sin^4 3x \cos^5 3x\, dx$

Transform by substitution

Let:

$u = 3x \qquad du = 3dx$

$\frac{du}{3} = dx$

apply changing limits: if $u = 3x$

if $x = 0, u = 3(0) = 0$

$x = \frac{\pi}{6}, \quad u = 3\left(\frac{\pi}{6}\right) = \frac{\pi}{2}$

$$\int_0^{\frac{\pi}{6}} \sin^4 x \cos^5 x\, dx$$

$= \int_{u_1}^{u_2} \sin^4 u \cos^5 u\, \frac{du}{3}$

$= \frac{1}{3}\int_0^{\frac{\pi}{2}} \sin^4 u \cos^5 u du$

$= \frac{1}{3} \cdot \frac{[(4-1)(4-3)(5-1)(5-3)]}{(4+5)(9-2)(9-4)(9-6)(9-8)} \cdot 1$

$= \frac{1}{3} \cdot \frac{(3\cdot1)(4\cdot2)}{(9\cdot7\cdot5\cdot3\cdot1)} \cdot 1$

$= \frac{24}{2835} \div 3$

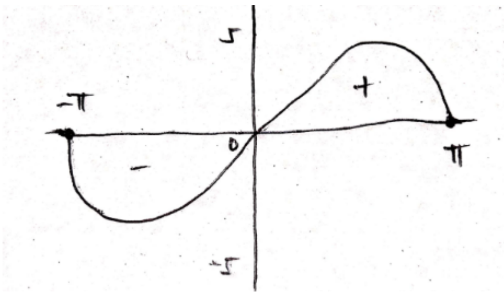
$= \frac{8}{945}$

ODD FUNCTION

6. $\int_{-\pi}^{\pi} - 5\, sin\, xdx$

= -5 (-cos x) $\int_{-\pi}^{\pi}$

= 5 cos x $\int_{-\pi}^{\pi}$



= $[5\, cos\, \pi] - [5\, cos\, (-\ \pi)]$
= -5 - (-5)
= 0

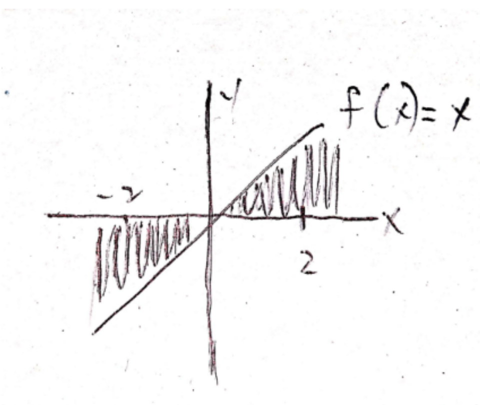
7. $\int_{-2}^{2} xdx$

$= \int x\, dx$

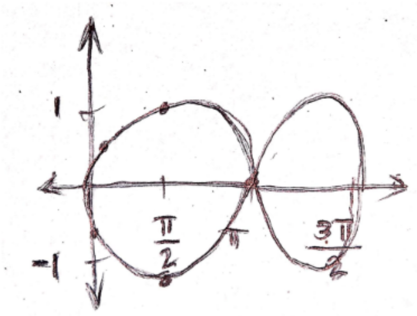
$= \frac{x^2}{2}$

$= \frac{x^2}{2} \int_{-2}^{2}$

$= \frac{x^2}{2} - \frac{(-2)^2}{2}$
= 0



8. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \sin x}{1+x^6} dx$



$f(x) = \frac{x^2 \sin x}{1+x^6}$

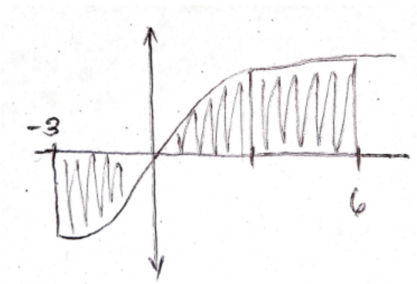
$f(-x) = \frac{(-x)^2 \sin(-x)}{1+(-x)^6}$

$= \frac{x^2(-\sin x)}{1+x^6}$

$= -[\frac{x^2 \sin x}{1+x^6}] = -f(x) = 0$

9. Find $\int_3^6 f(x)dx$ if $f(x)$is odd and $\int_3^6 f(x)dx = 10$

Since f is odd



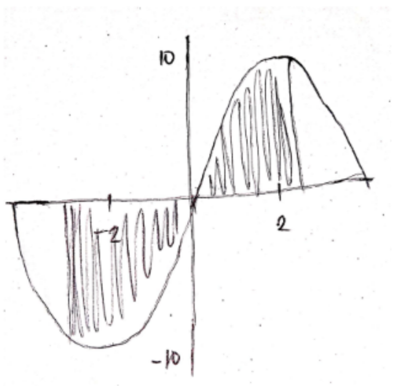
$\int_{-3}^{3} f(x)dx = 0$

$= \int_{-3}^{6} f(x)dx = \int_{-3}^{3} f + \int_{3}^{6} f = 10$

$= 0 + \int_3^6 f(x)dx = 10$

$\int_3^6 f(x)dx = 10$

10. $\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}}$ 9 sin(x)dx



$= \int 9\, sin(x)dx$

$= 9\int sin\,(x)dx$

$= 9(-\,cos\,(x))$

$= -\,9\,cos\,(x)$

$= -\,9\,cos(x)\ \int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}}$

$= -\,9cos(\frac{2\pi}{3}) - (9)cos(-\frac{2\pi}{3})$

$= 0$

EVEN FUNCTION

6. $\int_{-4}^{4} x^3 + 1 \, dx$

$= 4 \int_0^4 x^3 + 1$

$= 4 \left(\frac{1}{4} x^4 + x \right) \Big|_0^4$

$= 4 \left(\frac{1}{4} (4)^4 + 4 \right) - \cancel{\left(\frac{1}{4} (0)^4 + 0 \right)}$

$= 4 \left(\frac{1}{4} (256) + 4 \right)$

$= 272$

7. $\int_{-2}^{2} (x^2 + 2) \, dx$

$= 2 \int_0^2 (x^2 + 2) \, dx$

$= 2 \left[\frac{x^3}{3} + 2x \right] \Big|_0^2$

$= 2 \left[\frac{(2)^3}{3} + 2(2) - (0) \right]$

$= 2 \left(\frac{8}{3} + \frac{12}{3} \right)$

$= 2 \left(\frac{20}{3} \right)$

$= 40/3 = 13.33$

8. $\int_{-2}^{2} \left(3x^4 - \frac{1}{2} x^3 \right) dx$

$= 3 \int_{-2}^{2} x^4 dx - \cancel{\frac{1}{2} \int_{-2}^{2} x^3 dx}$

$= 3 \cdot 2 \int_0^2 x^4 \, dx = \frac{6}{5} x^5 \Big|_0^3$

$= \frac{6}{5} \cdot 3^5 = \frac{6}{5} \cdot 243$

$= \frac{1458}{5} = 291.6$

9. $\int_{-2}^{2} (3x^8 - 2x^2 + 3) \, dx$

$= 2 \int_0^2 (3x^8 - 2x^2 + 3) \, dx$

$= 2 \left[\frac{3x^9}{9} - \frac{2x^3}{3} + 3x \right] \Big|_0^2$

$= 2 \left(\left[\frac{3(2)^9}{9} - \frac{2(2)^3}{3} + 3(2) \right] - [0] \right)$

$= 2 \left(\frac{514}{3} \right)$

$= \frac{1028}{3} = 342.67$

10. $\int_{-2}^{2} x^6 + x^3 \, dx$

$= 2 \int_0^2 x^6 + x^3 \, dx$

$= 2 \left(\frac{x^7}{7} + \frac{x^4}{4} \right) \Big|_0^2$

$= 2 \left(\frac{2^7}{7} + \frac{2^4}{4} \right) - 0$

$= \frac{312}{7} = 44.57$

AREA BOUNDED BY TWO CURVES

TOP & BOTTOM

6.Find the area enclosed by the following two functions: $y=x^2$,$y=4x-x^2$.

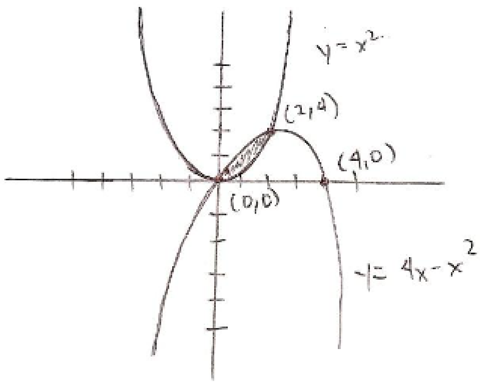
$y = x^2 \qquad y = 4x - x$

VERTEX: (2,4)

$x^2 = 4x - x^2$

$4x = 2x^2$

$x = 0, 2$



$A = \int_{x_1}^{x_2} (y_T - y_B)dx$

$A = \int_{0}^{2} (4x - x^2 - x^2)dx$

$= \int_{0}^{2} (4x - 2x^2)dx$

$= \frac{4x^2}{2} - \frac{2x^3}{3} \Big|_0^2$

$= \left[\frac{4(2)^2}{2} - \frac{2(2)^3}{3}\right] - \left[\frac{4(0)^2}{2} - \frac{2(0)^3}{3}\right]$

$= \frac{8}{3}$

7 Find the area between $y=4x^2$ and $y=5-x^2$ over the interval from -1 to 1.

$y = 4x^2 \qquad y = 5 - x^2$

$4x^2 = 5 - x^2$

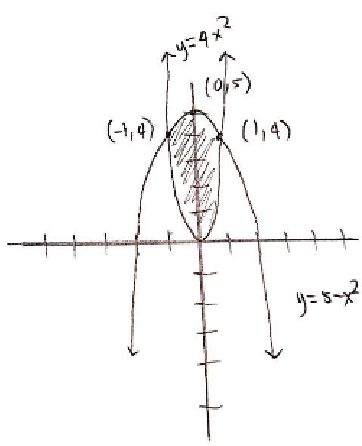
$5 = 4x^2 + x^2$

$5 = 5x^2$

$\frac{5}{5} = \frac{5x^2}{5}$

$\sqrt{1} = \sqrt{x^2}$

$x = 1, -1$



$A = \int_{x_1}^{x_2} (y_T - y_B)dx$

$= \int_{-1}^{1} \left(5 - x^2 - 4x^2\right)dx$

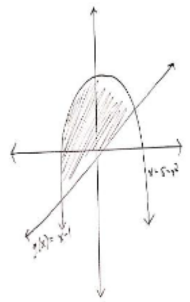
$= \int_{-1}^{1} \left(5 - 5x^2\right)dx$

$= 5x - \frac{5x^3}{3}\Big|_{-1}^{1}$

$= \left[5(1) - \frac{5(1)^3}{3}\right] - \left[5(-1) - \frac{5(-1)^3}{3}\right]$

$= \frac{20}{3}$

8 Find the area between the graphs of f(x) and g(x)=x-1 [-2,1].



$A = \int_{-2}^{1} \left(y_T - y_B\right)dx$

$= \int_{-2}^{1} \left(5 - x^2 - x + 1\right)dx$

$= \int_{-2}^{1} \left(-x^2 - x + 6\right)dx$

$= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 6x\right]_{-2}^{1}$

$= \left[-\frac{1^3}{3} - \frac{1^2}{2} + 6(1)\right] - \left[-\frac{(-2)^3}{3} - \frac{(-2)^2}{2} + 6(-2)\right]$

$= \frac{31}{6} - \left(-\frac{34}{3}\right)$

$= 16.5$

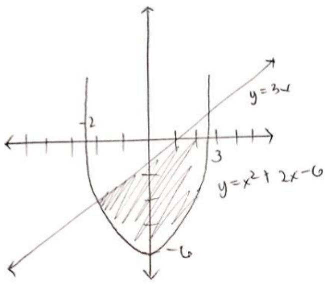
9 Find the area at the region bounded by $y=x^2+2x-6$ and y= 3x.

$x^2 + 2x - 6 = 3x$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3\,,\ x = -2$



$A = \int_{x_1}^{x_2} \left(y_T - y_B\right)dx$

$= \int_{-2}^{3} \left[3x - \left(x^2 + 2x - 6\right)\right]dx$

$= \int_{-2}^{3} \left[\left(3x - x^2 + 2x + 6\right)\right]dx$

$= \int_{-2}^{3} \left(-x^2 + x + 6\right)dx$

$= -\frac{x^3}{3} + \frac{x^2}{2} + 6x]\_ - 2\text{^}3$

$= \left[-\frac{3^3}{3} + \frac{3^2}{2} + 6(2)\right] - \left[-\frac{(-2)^3}{3} + \frac{(-2)^2}{2} + 6(-2)\right]$

$= \frac{125}{6}$

10 Find the area between y=3x²+5 and y= -x+3 over the interval from 0 to 2.

$$A = \int_{x_1}^{x_2} \left(y_T - y_B\right)dx$$

$$= \int_{0}^{2} \left[\left(3x^2 + 5\right) - (x + 3)\right]dx$$

$$= \int_{0}^{2} \left(3x^2 + x + 2\right)dx$$

$$= x^3 + \frac{x^2}{2} + 2x]\_0\text{^}2$$

$$= \left[2^3 + \frac{2^2}{2} + 2(2)\right] - [0]$$

$$= (8 + 2 + 4) - 0$$

$$= 14$$

RIGHT & LEFT

6. Calculate the area of the region bounded by $x = 4 + y^2, x = 3 - y^2,\ y =- 1$ and $y = 2$.

$$A = \int_{y1}^{y2} \left(x_T - x_B\right)dy$$

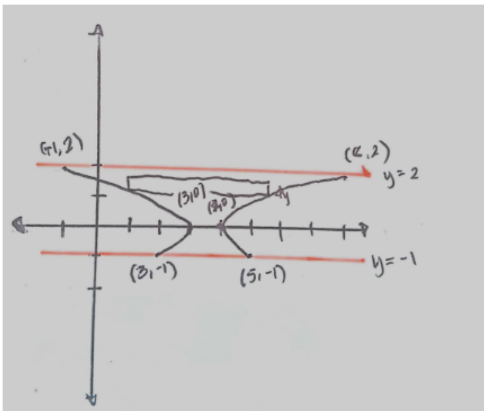
$$= \int_{-1}^{2} 4 + y^2 - \left(3 - y^2\right)dy$$

$$= \int_{-1}^{2} 1 + 2y^2 dy$$

$$= y + \frac{2y^3}{3}\Big|_{-1}^{2}$$

$$= \left[(2) + \frac{2(2)^3}{3}\right] - \left[(-1) + \frac{2(-1)^3}{3}\right]$$

$$= 9$$



7. Determine the region bounded by $x =- y^2 + 5$ and $x = (y - 1)^2$.

Find the intersection points.

$$- y^2 + 5 = (y - 1)^2$$

$$- y^2 + 5 = y^2 - 2y + 1$$

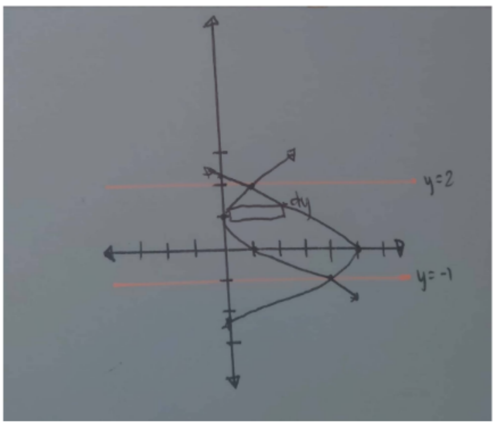
$$0 = 2y^2 - 2y - 4$$

$$0 = 2(y + 1)(y - 2)$$

Intersection is $y =- 1$ and $y = 2$.

$$A = \int_{y1}^{y2} \left(x_T - x_B\right)dy$$

$$= \int_{-1}^{2} - y^2 + 5 - (y - 1)^2 dy$$



$= -\frac{2y^3}{3} + \frac{2y^2}{2} + 4y|_{-1}^{2}$

$= -\frac{2y^3}{3} + y^2 + 4y|_{-1}^{2}$

$= \left[-\frac{2(2)^3}{3} + (2)^2 + 4(2)\right] - \left[-\frac{2(-1)^3}{3} + (-1)^2 + 4(-1)\right]$

$= 9$

8. Determine the area of the region bounded by $x = y^2 - 3y - 7$ and $x = y + 5$.

Find the intersection point.

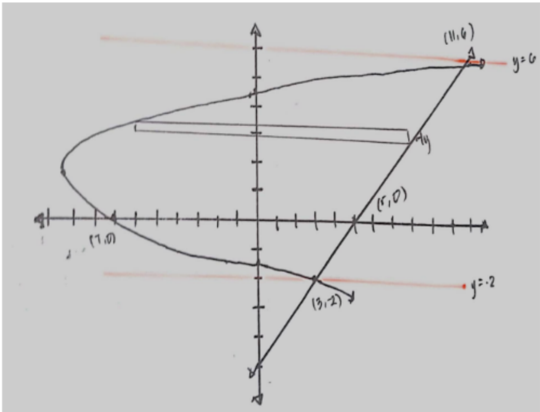
$y^2 - 3y - 7 = y + 5$

$y^2 - 4y - 12 = 0$

$(y - 6)(y + 2) = 0$

The intersection point is $y = 6$ and $y =- 2$

$A = \int_{y1}^{y2} \left(x_T - x_B\right) dy$



$= \int_{-2}^{6} y + 5 - \left(y^2 - 3y - 7\right) dy$

$= \int_{-2}^{6} - y^2 + 4y + 12 dy$

$=- \frac{y^3}{3} + \frac{4y^2}{2} + 12y|_{-2}^{6}$

$=- \frac{y^3}{3} + 2y^2 + 12y|_{-2}^{6}$

$= \left[-\frac{(6)^3}{3} + 2(6)^2 + 12(6)\right] - \left[-\frac{(-2)^3}{3} + 2(-2)^2 + 12(-2)\right]$

$= \frac{256}{3}$ *or* 85.3333333

9. Determine the area of the region bounded by $x = y^2 + 2$, $x = 6$, $y =- 4$ and $y = 4$.

$x = y^2 + 2 \qquad x = 6$

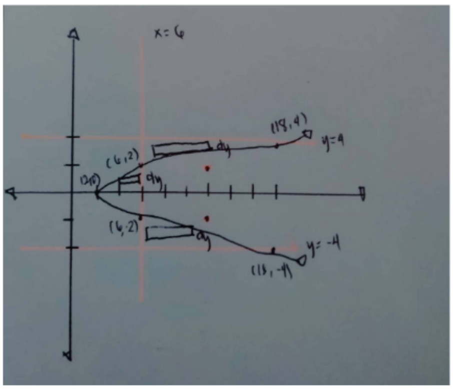
$6 = y^2 + 2$

$y^2 = 4$

$y = \pm 2$

$-4 \le x \le -2 \qquad -2 \le x \le 2 \qquad 2 \le x \le 4$

$A = \int_{y1}^{y2} \left(x_T - x_B\right) dy$



$= \int_{-4}^{-2} y^2 + 2 - 6\, dy + \int_{-2}^{2} 6 - \left(y^2 + 2\right) dy + \int_{2}^{4} y^2 + 2 - 6 dy$

$$= \int_{-4}^{-2} y^2 - 4\,dy + \int_{-2}^{2} 4 - y^2 dy + \int_{2}^{4} y^2 - 4dy$$

$$= \frac{y^3}{3} - 4y\Big|_{-4}^{-2} + \left(4y - \frac{y^3}{3}\right)\Big|_{-2}^{2} + \left(\frac{y^3}{3} - 4y\right)\Big|_{2}^{4}$$

$$= \left(\left[\frac{(-2)^3}{3} - 4(-2)\right] - \left[\frac{(-4)^3}{3} - 4(-4)\right]\right) + \left(\left[4(2) - \frac{(2)^3}{3}\right] - \left[4(-2) - \frac{(-2)^3}{3}\right]\right) + \left(\left[\frac{(4)^3}{3} - 4(4)\right] - \left[\frac{(2)^3}{3} - 4(2)\right.\right.$$

$$= \frac{32}{3} + \frac{32}{3} + \frac{32}{3}$$

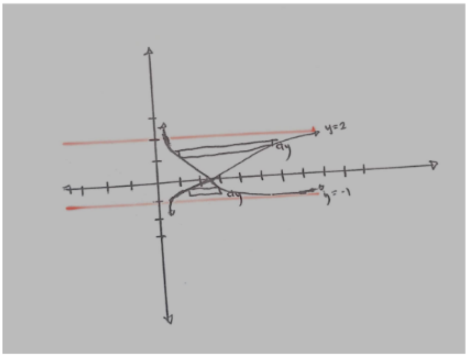
$$= 32$$

10. Determine the area of the region bounded by $x = e^{1+2y}, x = e^{1-y}, y = -1$ and $y = 2$

$e^{1+2y} = e^{1-y}$

$\frac{e^{1+2y}}{e^{1-y}}$

$y = 0$



$-1 \le x \le 0 \qquad 0 \le x \le 2$

$$A = \int_{y1}^{y2} \left(x_T - x_B\right) dy$$

$$= \int_{-1}^{0} e^{1-y} - e^{1+2y}\, dy + \int_{0}^{2} e^{1+2y} - e^{1-y}\, dy$$

$$= -e^{1-y} - \frac{e^{1+2y}}{2}\Big|_{-1}^{0} + \left(\frac{e^{1+2y}}{2} + e^{1-y}\right)\Big|_{0}^{2}$$

$$= \left(\left[-e^{1-y} - \frac{e^{1+2y}}{2}\right] - \left[-e^{1-y} - \frac{e^{1+2y}}{2}\right]\right) + \left(\left[\frac{e^{1+2y}}{2} + e^{1-y}\right] - \left[\frac{e^{1+2y}}{2} + e^{1-y}\right]\right)$$

$$= 3.495573076827804 + 70.49703624977118$$

$$= 73.9926$$

LOOP OF TWO CURVES

1. Find the area inside the graph of $r = 7 + 3\ \cos\theta$ and to the left of the $y - axis.$

$$A = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2}(7 + 3cos\theta)^2 d\theta$$

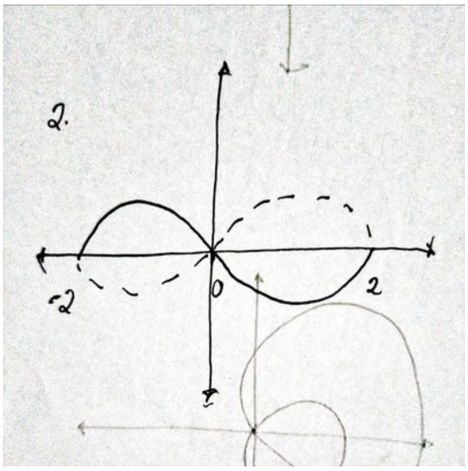
$$= \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 49 + 42cos\theta + 9\cos^2(\theta)d\theta$$

$$= \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 49 + 42cos\theta + \frac{9}{2}(1 + \cos(2\theta))d\theta$$

$$= \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{107}{2} + 42cos\theta + \frac{9}{2}(1 + \cos(2\theta))d\theta$$

$$= 42.0376\ sq.\ units$$

2. The area bounded by the loop of the curve $4y^2 = x^2 (4 - x^2)$ is



$$4y^2 = x^2(4 - x^2)$$

$$= \pm\frac{x}{2}\sqrt{(4 - x^2)}$$

$$\Rightarrow y = -\frac{x}{2}\sqrt{(4 - x^2)}$$

$$y = \frac{x}{2}\sqrt{(4 - x^2)}$$

$$y = -\frac{x}{2}\sqrt{(4 - x^2)},\ y = \frac{x}{2}\sqrt{(4 - x^2)}$$

$$A = 4 \bullet \int_0^2 \frac{x}{2}\sqrt{(4 - x^2)}dx$$

Let $4 - x^2 = t \Rightarrow -2xdx = dt$

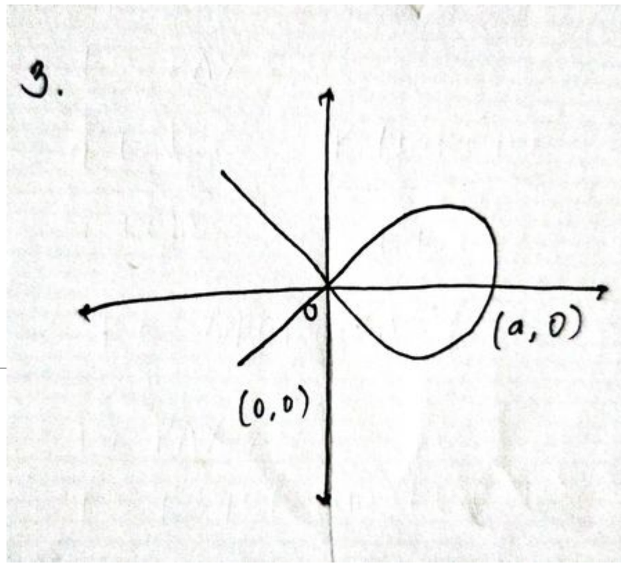
$$A = \int\_0\ \hat{}\ 4\sqrt{\{tdt\}}$$

$$= \left[\frac{t^{3/2}}{3/2}\right]$$

$$= \frac{2}{3}x[\sqrt{64} - 0]$$

$$= \frac{16}{3} sq.\,units$$

3. The area of the loop of the curve, $ay^2 = x^2(a - x)$ is



$$A = 2\int_0^a x\sqrt{\frac{a - x}{a}}\ dx$$

$$\sqrt{\frac{a - x}{a}} = t \Rightarrow -\frac{x}{a} = t^2 \Rightarrow x = a(1 - t^2)$$

$$A = 2\int_1^0 a(1 - t^2)t(-2at)dt$$

$$= 4a^2 \int_0^1 (t^2 - t^4)dt$$

$$= 4a^2\left[\frac{t^3}{3} - \frac{t^5}{5}\right]$$

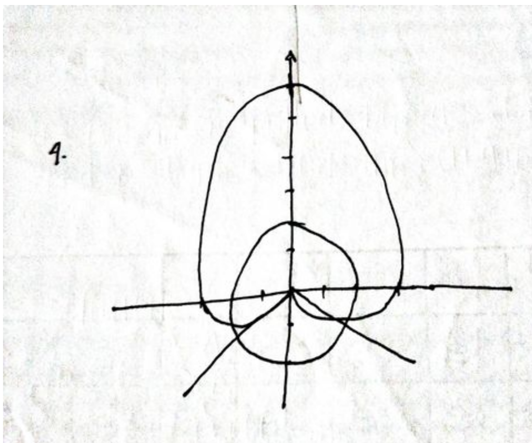
$$= 4a^2\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$= \frac{8a^2}{16} sq.\ \ units$$

4. Find the area that is inside $r = 3 + 3\sin\theta$ and outside $r = 2$

$$sin\theta = -\frac{1}{3} \Rightarrow \theta = \sin^{-1}\left(-\frac{1}{3}\right) = -0.3398$$

$$\theta = \pi + 0.3398 = 3.4814$$



$$A = \int_{-0.3398}^{3.4814} \frac{1}{2}\left[(3 + 3sin\theta)^2 - (2)^2\right]d\theta$$

$$= \frac{1}{2}\int_{-0.3398}^{3.4814} 5 + 18sin\theta + 9\sin^2(\theta)d\theta$$

$$= \frac{1}{2}\int_{-0.3398}^{3.4814} 5 + 18sin\theta + \frac{9}{2}(1 - \cos(2\theta))d\theta$$

$$= \frac{1}{2}\int_{-0.3398}^{3.4814} \frac{19}{2} + 18sin\theta - \frac{9}{2}\cos(2\theta)d\theta$$

$$= 33.7074 sq.\ \ units$$

5. Find the area inside the inner loop of $r = 3 - 8\,cos\theta$

$3 - 8\,cos\theta = 0$

$cos\theta = \frac{3}{8} \Rightarrow \theta = \cos^{-1}\left(\frac{3}{8}\right) = 1.1864$

$\theta = 2\pi - 1.1864 = 5.0968\theta = -1.1864$

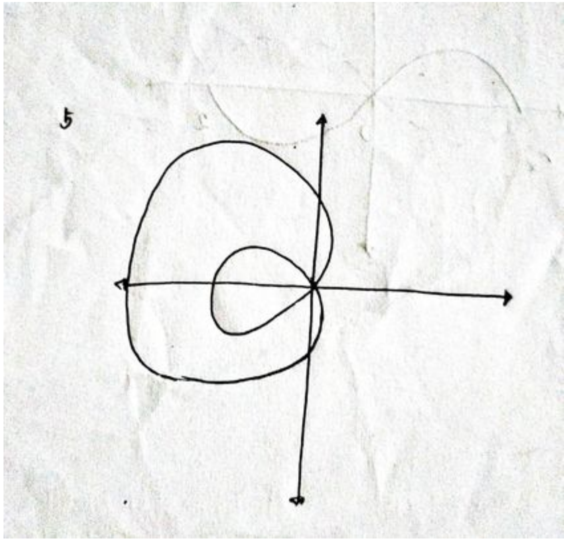
$$A = \int_{-1.1864}^{1.1864} \frac{1}{2}(3 - 8cos\theta)^2 d\theta$$

$$= \frac{1}{2}\int_{-1.1864}^{1.1864} 9 - 48cos\theta + 64\cos^2(\theta)d\theta$$

$$= \frac{1}{2}\int_{-1.1864}^{1.1864} 9 - 48cos\theta + 32(1 + \cos(2\theta))d\theta$$

$$= \frac{1}{2}\int_{-1.1864}^{1.1864} 41 - 48cos\theta + 32\cos(2\theta)d\theta$$

$= 15.2695\ sq.\ \ units$



ARC LENGTH

6. Find the length of the line segment given by the equation $y = 8x + 4$ from $x = 2\ to\ x = 5$.

$y = 8x + 4 \qquad \frac{dy}{dx} = 8 \ \left(\frac{dy}{dx}\right)^2 = 64$

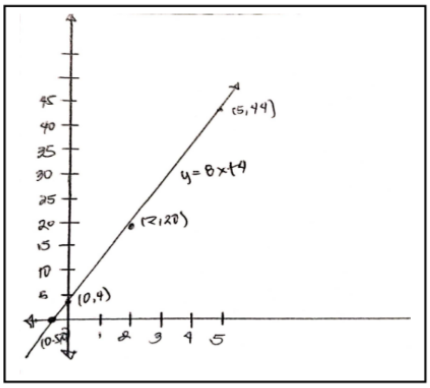
$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_2^5 \sqrt{1 + 64}dx$

$= \sqrt{65}x\Big|_2^5$

$= [\sqrt{65}(5)] - [\sqrt{65}(2)]$

$= 24.18677\ units$

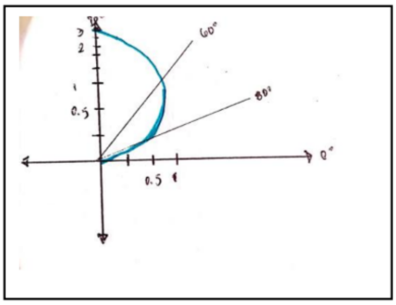


7. Find the length of the curve $r = 3(1 - \cos\theta)$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$.

Solution:

θ	r
0°	0
30°	0.402
60°	1.5
90°	3

$S = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$



$r = 3(1 - \cos\theta)$; $\qquad r^2 = 9(1 - \cos\theta)^2 = 9(1 - 2\cos\theta + \cos^2\theta)$

$\frac{dr}{d\theta} = 3(\sin\theta) \qquad \left(\frac{dr}{d\theta}\right)^2 = 9\sin^2\theta$

$S = \int_0^{\frac{\pi}{2}} \sqrt{9 - 18\cos\theta + 9\cos^2\theta + 9\sin^2\theta} \cdot d\theta$

$= \int_0^{\frac{\pi}{2}} \sqrt{9 - 18\cos\theta + 9(1)} \cdot d\theta$

$= \int_0^{\frac{\pi}{2}} \sqrt{18 - 18\cos\theta} \cdot d\theta$

$= -3.5147\ units$

8. Determine the length of $y = 2 + 4x^{\frac{3}{2}}, \quad 0 \le x \le 3$.

$y = 2 + 4x^{\frac{3}{2}} \qquad \frac{dy}{dx} = 6x^{\frac{1}{2}} \qquad \left(\frac{dy}{dx}\right)^2 = 36x$

$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx$

$= \int_0^3 \sqrt{1+36x} \cdot dx \qquad u = 1 + 36x \qquad du = 36x \qquad dx = \frac{du}{36}$

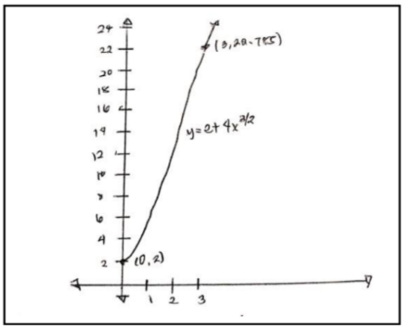
$= \int_1^{109} \sqrt{u}\,\frac{du}{36} \qquad u = 1 + 36(0) = 1 \qquad u = 1 + 36(3) = 109$

$= \frac{1}{36}\int_1^{109} u^{\frac{1}{2}} du$

$= \frac{1}{36} \cdot u^{\frac{3}{2}} \cdot \frac{2}{3}\Big|_1^{109}$

$= \frac{2}{108}\left[109^{\frac{3}{2}} - 1^{\frac{3}{2}}\right]$

$= 21.0554\ units$



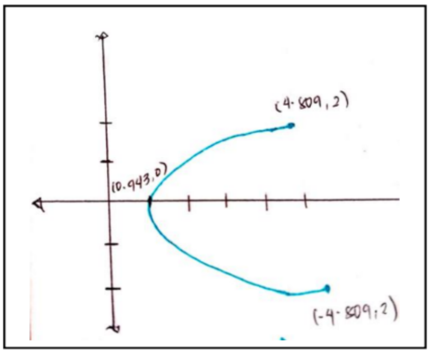
9. Determine the length of the curve $x = \frac{1}{3}(y^2+2)^{\frac{3}{2}}$, $1 \le y \le 2$.

$x = \frac{1}{3}(y^2+4)^{\frac{3}{2}} \qquad \frac{dx}{dy} = y\sqrt{y^2+2} \qquad \left(\frac{dx}{dy}\right)^2 = y^2(y^2+2) = y^4 + 2y^2$

$\int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$\int_1^2 \sqrt{1 + y^4 + 2y^2}\, dy$

$= \int_1^2 \sqrt{(y^2+1)^2}\, dy$



$= \int_1^2 (y^2+1) dy$

$= \frac{y^3}{3} + y\Big|_1^2$

$= \left(\frac{(2)^3}{3} + 2\right) - \left(\frac{(1)^3}{3} + 1\right)$

$= \frac{14}{3} - \frac{4}{3}$

$= 3.33333333333\ units$

10.Find the length of the curve $x = \frac{1}{2}\sqrt{y}\,(y-2)$ from $y = 0$ to $y = 3$.

$x = \frac{1}{2}\sqrt{y}\,(y-2)$

$= \frac{1}{2}y^{\frac{1}{2}}(y-2)$

$= \frac{1}{2}y^{\frac{3}{2}} - y^{\frac{1}{2}}$

$\frac{dx}{dy} = \frac{1}{2}\cdot\frac{3}{2}y^{\frac{1}{2}} - \frac{1}{2}y^{-\frac{1}{2}}$

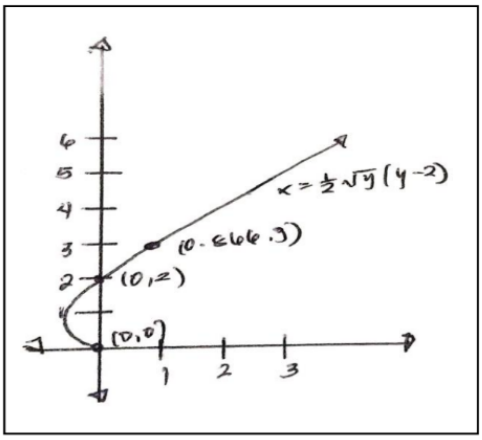
$\frac{dx}{dy} = \frac{1}{2}y^{\frac{1}{2}} - \frac{1}{2}y^{-\frac{1}{2}}$

$\left(\frac{dx}{dy}\right)^2 = (\frac{1}{2}y^{-\frac{1}{2}}(y' - 1))^2$

$\left(\frac{dx}{dy}\right)^2 = \frac{1}{4}y^{-1}(y-1)^2$

$\int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_0^3 \sqrt{1 + \frac{1}{4}y^{-1}(y-1)^2}\, dy$



$= \int_0^3 \sqrt{\frac{1}{4}y^{-1}(4y + (y-1)^2)}\, dy$

$= \int_0^3 \sqrt{\frac{1}{4}y^{-1}\left(4y + (y^2 - 2y + 1)\right)}\, dy$

$= \int_0^3 \sqrt{\frac{1}{4}y^{-1}(y^2 - 2y + 1)}\, dy$

$= \frac{1}{2}\int_0^3 y^{-\frac{1}{2}}(y+1)dy$

$= \frac{1}{2}\int_0^3 (y^{\frac{1}{2}} + y^{-\frac{1}{2}})\, dy$

$= \frac{1}{2}\left[\frac{2}{3}y^{\frac{3}{2}} + y^{\frac{1}{2}} \cdot \frac{2}{1}\right]\Big|_0^3$

$= \frac{1}{3}y^{\frac{3}{2}} + y^{\frac{1}{2}}\Big|_0^3$

$= (\frac{1}{3}(3)^{\frac{3}{2}} + (3)^{\frac{3}{2}}) \; - (\frac{1}{3}(0)^{\frac{3}{2}} + (0)^{\frac{3}{2}})$

$= 3.4641\ units$

PAPPUS THEOREM

1. $A = \int_0^4 (4x - x^2)dx$

$= \left[2x^2 - \frac{1}{3}x^3\right]$

$= 32 - \frac{64}{3}$

$= \frac{32}{3}$

$\bar{y}A = \int_0^4 \frac{1}{2}(4x - x^2)^2 dx$

$= \int_0^4 \frac{1}{2}(16x^2 - 8x^3 + x^4)dx$

$= \frac{256}{15}$

$\bar{y} = \frac{\frac{256}{15}}{\frac{32}{3}}$

$= \frac{8}{5} \quad (\overline{x}, \overline{y}) = \left(2,\ \frac{8}{5}\right)$

2. $\bar{y}A = \int_{-R}^{R} \frac{1}{2}\left(\sqrt{R^2 - x^2}\right)^2 dx$

$= 2\int_0^R \frac{1}{2}(R^2 - x^2)dx$

$= R^3 - \frac{1}{3}R^3 = \frac{2}{3}R^3$

$\bar{y} = \frac{\frac{2}{3}R^3}{\frac{1}{2}\pi R^2}$

$(\overline{x}, \overline{y}) = \left(0,\ \frac{4R}{3\pi}\right)$

$V = 2\pi\bar{y}A$

$= 2\pi\left(\frac{4R}{3\pi}\right)\left(\frac{\pi R^2}{2}\right)$

$= \frac{4}{3}\pi R^3$

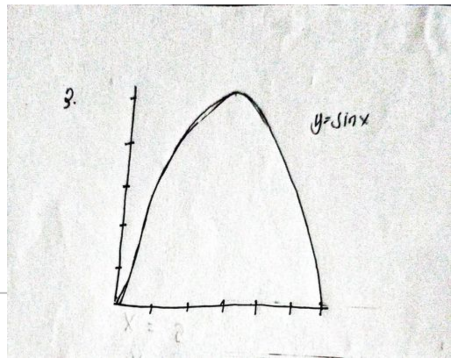
3. $A = \int_0^{\pi} \sin x dx = [-\cos x] = 2$

$\bar{y}A = \int_0^{\pi} \frac{1}{2}\sin^2 x dx$

$\bar{y}A = \int_0^{\pi} \frac{1}{2} \bullet \frac{1 - \cos 2x}{2} dx$

$= \frac{\pi}{4}$

$\bar{y} = \frac{\frac{\pi}{4}}{2} = \frac{\pi}{8} \quad (\overline{x}, \overline{y}) = \left(\frac{\pi}{2}, \frac{\pi}{8}\right)$



4. Find the intersection of the curves

$y = \frac{1}{2}x^2$

$y = \frac{1}{2}\left(\frac{1}{2}y^2\right)^2$

$y = \frac{1}{8}y^4$

$8y = y^4$

$0 = y^4 - 8y$

$0 = y(y^3 - 8)$

$y = 0 \ or \ y = 2$

$A = \int_0^2 \left(\sqrt{2x} - \frac{1}{2}x^2\right)dx$

$= \frac{2\sqrt{2}}{3} \bullet 2\sqrt{2} - \frac{4}{3}$

$= \frac{8}{3} - \frac{4}{3}$

$= \frac{4}{3}$

$\bar{y}A = \int_0^2 \frac{1}{2}\left[(\sqrt{2x})^2 - \left(\frac{1}{2}x^2\right)^2\right]dx$

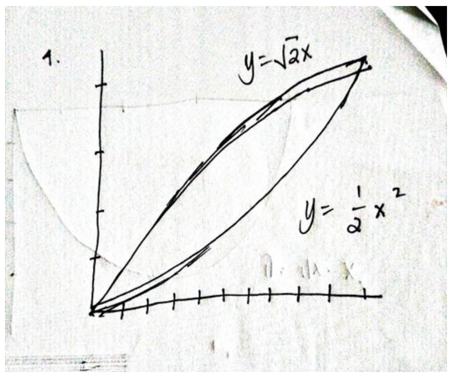
$= \int_0^2 \frac{1}{2}\left(2x - \frac{1}{4}x^4\right)dx$

$= \frac{1}{2}\left(4 - \frac{32}{20}\right)$

$= \frac{6}{5}$

$\bar{y} = \frac{\frac{6}{5}}{\frac{4}{3}} = \frac{9}{10}$

$(\overline{x}, \overline{y}) = \left(\frac{9}{10}, \frac{9}{10}\right)$



5. Find the points of intersection of the curves

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x + 1 = 0 \ or \ x - 2 = 0$

$x = -1 \ or \ x = 2$

$A = \int_{-1}^{2} \left[(x+2) - x^2\right] dx$

$= \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$

$= \frac{9}{2}$

$\overline{x}A = \int_{-1}^{2} x\left(x + 2 - x^2\right) dx$

$\overline{x}A = \int_{-1}^{2} \left(x^2 + 2x - x^3\right) dx$

$= \left(\frac{8}{3} + 4 - 4\right) - \left(-\frac{1}{3} + 1 - \frac{1}{4}\right)$

$= \frac{9}{4}$

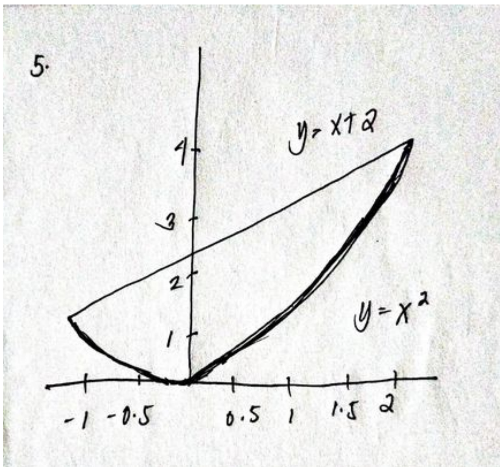
$\overline{y}A = \int_{-1}^{2} \frac{1}{2}\left[(x+2)^2 - \left(x^2\right)^2\right] dx$

$\overline{y}A = \int_{-1}^{2} \frac{1}{2}\left(x^2 + 4x + 4 - x^4\right) dx$

$\overline{y}A = \frac{1}{2}\left[\left(\frac{8}{3} + 8 + 8 - \frac{32}{5}\right) - \left(-\frac{1}{3} + 2 - 4 + \frac{1}{5}\right)\right]$

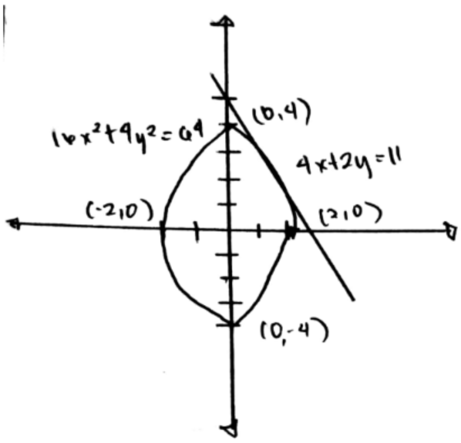
$= \frac{36}{5}$

$\overline{x} = \frac{\frac{9}{4}}{\frac{9}{2}} = \frac{1}{2}, \ \overline{y} = \frac{\frac{36}{5}}{\frac{9}{2}} = \frac{8}{5} \qquad (\overline{x}, \overline{y}) = \left(\frac{1}{2}, \frac{8}{5}\right)$



PAPPUS THEOREM 2

6. Determine the volume generated by rotating the curve $16x^2 + 4y^2 = 64$ about the line $4x + 2y = 11$.



$16x^2 + 4y^2 = 64$

$\frac{x^2}{4} + \frac{y^2}{16} = 1$

$a = 2$

$b = 4$

$V = A \times 2\pi z$

$A = \pi ab \qquad z = \frac{Ax + By + c}{\pm\sqrt{A^2 + B^2}}$

$= \pi(2)(4) \qquad = \frac{4(0) + 2(0) - 11}{\pm\sqrt{4^2 + 2^2}}$

$= 8\pi \qquad = 2.45967477525$

$V = A \times 2\pi z$

$= (8\pi) \times 2\pi(2.45967477525)$

$= 388.42$

7. Determine the volume of the solid obtained by rotating the region bounded by $x = (y - 4)2$, the x-axis and the y-axis about the x-axis.

$x = (y - 4)2$

$radius = y$

$width = (y - 4)2$

$Ay = 2\pi(radius)(width)$

$Ay = 2\pi(y)(y - 4)2$

$Ay = 2\pi(y^3 - 8y^2 + 16y)$

$y = 0 \qquad y = 4$

$V = cdAydy$

$V = 042\pi(y^3 - 8y^2 + 16y)dy$

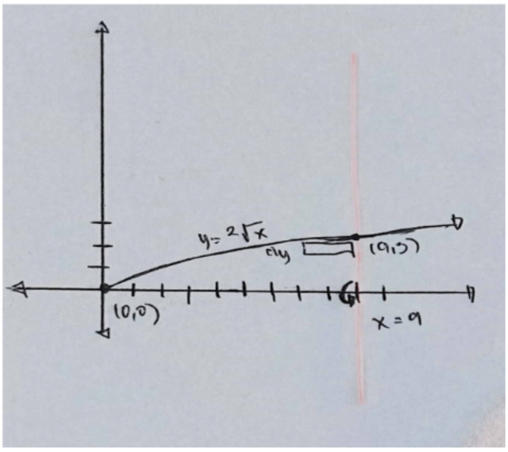
$= 2\pi y44 - 8y33 + 16y2204$

$= 2\pi y44 - 8y33 + 8y204$

$= 2\pi(4)44 - 8(4)33 + 8(4)2 - 2\pi(0)44 - 8(0)33 + 8(0)2$

$= 134.0412\ cu.\ units$

8. Determine the volume of the solid obtained by rotating the region bounded by $y = \sqrt[2]{x}$, $x = 9$ and the x-axis about the x-axis.



$y = \sqrt[2]{x}$

$x = y^2$

$A(y) = 2\pi(r)(z)$

$= 2\pi(y)(9 - y^2)$

$= 2\pi(9y - y^3)$

$V = \int_c^d A(y)dy$

$= \int_0^3 2\pi\left(9y - y^3\right)dy$

$= 2\pi\left(\frac{9y^2}{2} - \frac{y^4}{4}\right)\Big|_0^3$

$= \left[2\pi\left(\frac{9(3)^2}{2} - \frac{(3)^4}{4}\right)\right] - \left[2\pi\left(\frac{9(0)^2}{2} - \frac{(0)^4}{4}\right)\right]$

$= 127.2345$

9. Determine the volume of the solid obtained by rotating the region bounded by $y = 9x$ and $y = x^3$ about the y-axis. For this problem assume that $x \geq 0$.

$x^3 = 9x \qquad x^3 - 9x = 0$

$xx2 - 9 = 0$

$x = 0 \qquad x = +- 3$

$radius = x \qquad height = x^3 - 9x$

$Ax = 2\pi(radius)(height)$

$Ax = 2\pi(x)(x^3 - 9x)$

$Ax = 2\pi(x^4 - 9x^2)$

$V = abAxdx$

$= 032\pi(x^4 - 9x^2)$

$= 2\pi(x55 - 9x33)03$

$= 2\pi(3)55 - 9(3)33 - 2\pi(0)55 - 9(0)33$

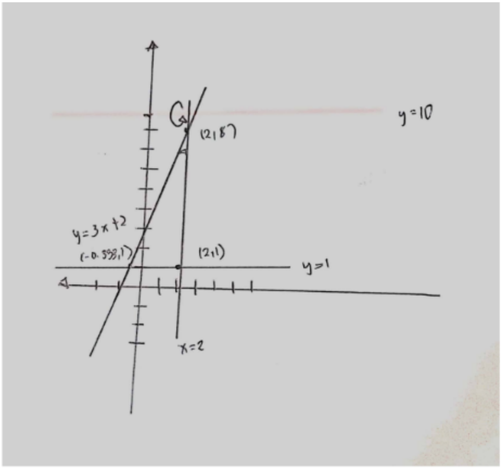
$=- 203.5752\ cu.\ units$

10. Determine the volume of the solid obtained by rotating the region bounded by $y = 3x + 2, y = 1$ and $x = 2$ about the line $y = 10$.

$Radius = 10 - y \qquad Width = 2 - \frac{1}{3}(y - 2)$

$= \frac{8}{3} - \frac{1}{3}y$

$A(y) = 2\pi(r)(z)$



$= 2\pi(10 - y)\left(\frac{8}{3} - \frac{1}{3}y\right)$

$= 2\pi(10 - y)\left(\frac{8}{3} - \frac{1}{3}y\right)$

$= 2\pi(\frac{1}{3})(80 - 18y + y^2)$

$V = \int_c^d A(y)dy$

$= \int_1^8 2\pi(\frac{1}{3})(80 - 18y + y^2)dy$

$= 2\pi\left(\frac{1}{3}\right)\left[80y - \frac{18y^2}{2} + \frac{y^3}{3}\right]\Big|_1^8$

$= \left[2\pi\left(\frac{1}{3}\right)\left[80(8) - \frac{18(8)^2}{2} + \frac{(8)^3}{3}\right]\right] - \left[2\pi\left(\frac{1}{3}\right)\left[80(1) - \frac{18(1)^2}{2} + \frac{(1)^3}{3}\right]\right]$

$= \frac{980\pi}{9}$

CIRCULAR DISK METHOD

6. Given the area in the first quadrant bounded by $x^2=6y$, the line y-2=0 and the y-axis. What is the volume generated when the area is revolved about the line y-2=0.

$x^2 = 6y \qquad y - 2 = 0$

$y = \frac{x^2}{6} \qquad y = 2$

$Yu - Y1 = 2 - \frac{x^2}{6} \qquad X1 = 0$

$x^2 = 6(2) \qquad x = \pm\sqrt{12} \qquad x2 = \pm\sqrt{12}$

$$V = \pi \int_{x_1}^{x_2} \left(y_u - y_l\right)^2 dx$$

$$= \pi \int_0^{\sqrt{12}} \left(2 - \frac{x^2}{6}\right)^2 dx$$

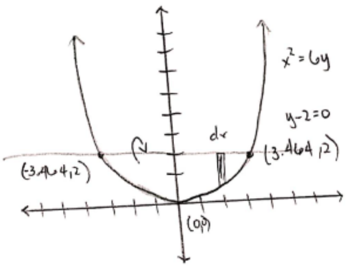
$$= \pi \int_0^{\sqrt{12}} \left(\frac{12-x^2}{6}\right)^2 dx$$

$$= \frac{\pi}{36} \int_0^{\sqrt{12}} \left(12^2 - 24x^2 + x^4\right) dx$$

$$= \frac{\pi}{36}\left[12^2 x - \frac{24x^3}{3} + \frac{x^5}{5}\right]\Big|_0^{\sqrt{12}}$$

$$= \left[\frac{\pi}{36}\left[12^2\left(\sqrt{12}\right) - \frac{24\left(\sqrt{12}\right)^3}{3} + \frac{\left(\sqrt{12}\right)^5}{5}\right]\right] - \left[\frac{\pi}{36}\left[12^2(0) - \frac{24(0)^3}{3} + \frac{(0)^5}{5}\right]\right]$$

$$= 23.2166$$



7. The area bounded by the curves $x^2=y$ and the line $x=4$ and the x-axis in the first quadrant is revolved about the y-axis. What is the volume generated?

$x^2 = y$	$x = 4$	$4^2 = y$
$x = \sqrt{y}$	$x^2 = y$	$16 = y$

$$V = \pi \int_{y_1}^{y_2} \left(x_R - x_l\right)^2 dy$$

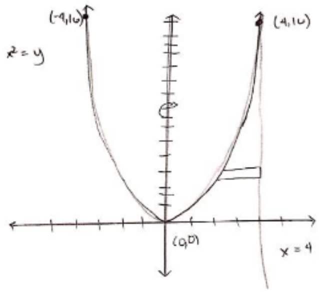
$$= \pi \int_0^{16} \left(4 - \sqrt{y}\right)^2 dy$$

$$= \pi \int_0^{16} 16 - y\, dy$$

$$= \pi\left[16y - \frac{y^2}{2}\right]\Big|_0^{16}$$

$$= \left[\pi\left[16(16) - \frac{(16)^2}{2}\right]\right] - \left[\pi\left[16(0) - \frac{(0)^2}{2}\right]\right]$$

$$= 402.1238$$



8 Determine the volume of the solid obtained by rotating the region bounded by $x=y^2-6y+12$ and $x=7$ about the y-axis.

$$x = y^2 - 6y + 12$$

$$7 = y^2 - 6y + 12$$

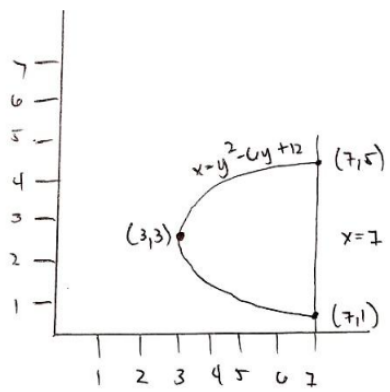
$$y^2 - 6y + 12 - 7 = 0$$

$$y^2 - 6y + 5 = 0$$

$$(y - 5)(y - 1)$$

$$y = 5,\ y = 1$$

$$(7, 1),\ (7, 5)$$



$$V = \pi\int_{y_1}^{y_2} \left(x_R - x_l\right)^2 dy$$

$$= \pi\int_{1}^{5} (17)^2 - \left(y^2 - 6y + 12\right)^2$$

$$= \pi\int_{1}^{5}\left[49 - \left(y^4 - 12y^3 + 60y^2 - 144y + 144\right)\right]$$

$$= \pi\int_{1}^{5} - y^4 + 12y^3 - 60y^2 + 144y - 95$$

$$= \pi\int_{1}^{5}(- \tfrac{y^5}{5} + \tfrac{12y^4}{4} - \tfrac{60y3}{3} + 144y^2 - 95y)]\ \_1\text{^}5$$

$$= \pi(- \tfrac{y^5}{5} + 3y^4 - 20y^3 + 72y^2 - 95y)]\_1\text{^}5$$

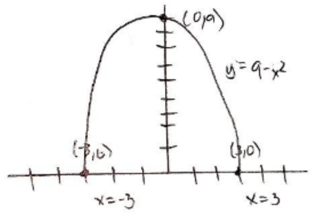
$$= [\pi(\tfrac{(-5)^5}{5} + 3(5)^4 - 20(5)^3 + 72(5)^2 - 95(5)] - [\pi(- \tfrac{(1)^3}{5} + 3(1)^4 - 20(1)^3 + 72(1)^2 - 95(1)]$$

$$= \tfrac{5767}{5} or\ 361.9115$$

9. Determine the volume of the solid by rotating the region bounded by $y=9-x^2$, $x=-3$, $x=3$ and the x-axis about the x-axis.

$$y = 9 - x^2$$

$$V = \pi\int_{x_1}^{x_2} y^2 dx$$



$$= \pi\int_{-3}^{3}\left(9 - x^2\right)dx$$

$$= \pi\int_{-3}^{3}\left(81 - 18x^2 + x^4\right)dx$$

$$= \pi(81x - \tfrac{18x^3}{3} + \tfrac{x^5}{5})]\_ - 3\text{^}3$$

$$= \pi(81x - 6(3)^3 + \tfrac{x^5}{5})\,]_{-3}^{3}$$

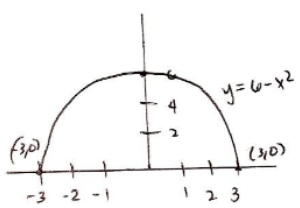
$$= \left[\pi\left(81(3) - 6(3)^3 + \tfrac{(-3)^5}{5}\right)\right] - \left[\pi\left(81(-3) - 6(-3)^3 + \tfrac{(-3)^5}{5}\right)\right]$$

$$= \tfrac{1296\pi}{5}\ or\ 814.3005$$

10 Determine the volume of the solid obtained by rotating the region bounded by $y= 6-x^2$, $x=3$, $x=-3$ and the x axis about the x axis.

$$y = 6 - x^2$$

$$V = \pi\int_{x_1}^{x_2} y^2 dx$$



$$= \pi\int_{-3}^{3}\left(6 - x^2\right)dx$$

$$= \pi\int_{-3}^{3}\left(36 - 12x^2 + x^4\right)dx$$

$$= \pi\left(36x - \tfrac{12x^3}{3} + \tfrac{x^5}{5}\right)|\_ - 3\text{^}3$$

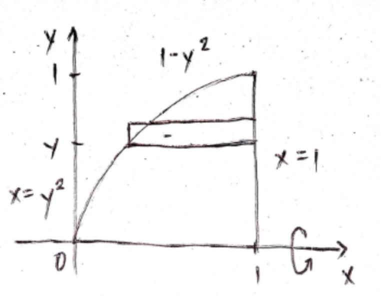
$$= \pi\left(36x - 4x^3 + \tfrac{x^5}{5}\right)]\_ - 3\text{^}3$$

$$= \left[\pi\left(36(3) - 4(3)^3 + \tfrac{(-3)^5}{5}\right)\right] - \left[\pi\left(36(-3) - 4(-3)^3 + \tfrac{(-3)^5}{5}\right)\right]$$

$$= \left(\tfrac{966\pi}{5}\right)or\ 193.2$$

CYLINDRICAL SHELL METHOD

6. Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.

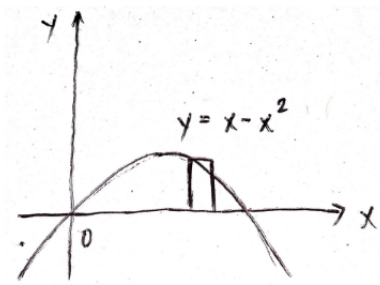


$$V = \int_0^1 (2\pi y)(1 - y^2) = 2\pi \int_0^1 (y - y^2)dy$$

$$= 2\pi[\frac{y^2}{2} - \frac{y^4}{4}] \int_0^1$$

$$= \frac{\pi}{2}$$

7. Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.



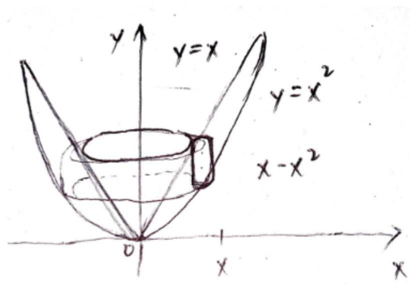
$$V = \int_0^1 2\pi(2 - x)(x - x^2)dx$$

$$= \int_0^1 2\pi(x^3 - 3x^2 + 2x)dx$$

$$= 2\pi[\frac{x^4}{4} - x^3 + x^2]$$

$$= \frac{\pi}{2}$$

8. Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.



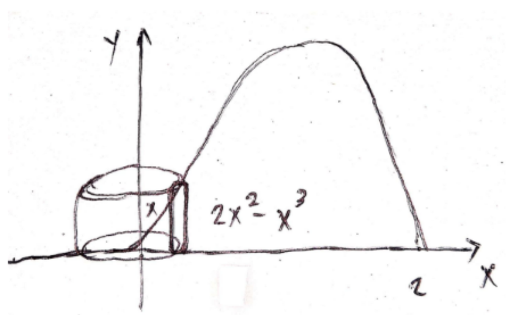
$$V = \int_0^1 (2\pi x)(x - x^2)dx$$

$$= 2\pi \int_0^1 (x^2 - x^3)dx$$

$$= 2\pi[\frac{x^3}{3} - \frac{x^4}{4} \int_0^1$$

$$= \frac{\pi}{6}$$

9. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.



$$V = \int_0^2 (2\pi x)(2x^2 - x^3)dx$$

$$= 2\pi \int_0^2 (2x^3 - x^4)dx$$

$$= 2\pi[\frac{1}{2}x^4 - \frac{1}{5}x^5 \int_0^2$$

$$= 2\pi(8 - \frac{32}{5})$$

$$= \frac{16}{5}\pi$$

10. Find the volume of the solid generated by revolving the triangular region bounded by $y = x, y = 0$, and $x = 2$ about the line $x = 3$.

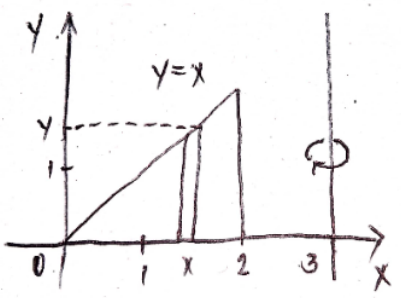
$$V = 2\pi \int_0^2 (3 - x)x\,dx$$

$$= 2\pi \int_0^2 (3x - x^2)dx$$

$$= 2\pi[\tfrac{3}{2}x^2 - \tfrac{1}{3}x^3 \int_0^2$$

$$= 2\pi[[\tfrac{3}{2}(2)^2 - \tfrac{1}{3}(2)^3] - 0]$$

$$= \frac{20\pi}{3}$$



VARIGNON'S THEOREM

1.

$$M_{xz} = \int Y_c dV \quad ; \quad \bar{y} = \frac{M_{xz}}{V}$$

x	y
0	0
9	3

$$V = 2\pi \int_0^9 xydx \quad = 2\pi \int_0^9 x(3 - y)dx$$

$$M_{xz} = \int Y_c dV \; = 2\pi \int_0^9 \left(\frac{3+y}{2}\right)(x)(\sqrt{x})dx$$

$$M_{xz} = 381.70$$

$$\bar{y} = \frac{M_{xz}}{V} = \frac{381.70}{152.68}$$

$$\bar{y} = 2.5$$

2. $A = \int_0^a (\sqrt{kx})dx$

$$A = \sqrt{k} \int_0^a \sqrt{x}dx$$

$$= \int_0^a \frac{1}{2}y^2 dx$$

$$= \frac{1}{2}\;k \int_0^a xdx$$

$$= \frac{b^2}{2a}\left[\frac{a^2}{2}\right]$$

$$= \sqrt{k} \int_0^a x\sqrt{x}dx$$

$$= \frac{2a^2b}{5}$$

$$\bar{x} = \left(\frac{\frac{2a^2b}{5}}{\frac{2ab}{3}}\right) or \; \bar{x} = \frac{3a}{5}$$

$$\bar{y} = \frac{3b}{8}$$

3. *Solving for A*

$$dA = ydx$$

$$\int_0^6 dA = \int_0^6 \left(3 - \frac{x}{2}\right)dx$$

$$A = [3x - \frac{x^2}{4}]_0^6$$

$$A = \left[3(6) - \frac{36}{4}\right]$$

$$\boldsymbol{A = 9\;sq.units}$$

Solving for $\bar{x}$

$$A\bar{x} = \int_0^6 Xc\;dA$$

$$A\bar{x} = \int_0^6 x\left(3 - \frac{x^2}{2}\right)dx$$

$$A\bar{x} = \int_0^6 \left(3x - \frac{x^2}{2}\right)dx$$

$$A\bar{x} = [\frac{3x^2}{2} - \frac{x^3}{3}]_0^6$$

$$9\bar{x} = 18$$

$$\boldsymbol{\bar{x} = 2\;units}$$

Solving for $\bar{y}$

$$A\bar{y} = \int_0^6 Yc\;dA$$

$$A\bar{y} = \frac{1}{2}\int_0^6 (3 - \frac{x}{2})\,(3 - \frac{x}{2})dx$$

$$A\bar{y} = \frac{1}{2}\int_0^6 (9 - 3x + \frac{x^2}{4})dx$$

$$A\bar{y} = \frac{1}{2}[9x - \frac{3}{2}x^2 + \frac{x^3}{12}]_0^6$$

$$\bar{y} = \frac{1}{3}(3)$$

$$\boldsymbol{\bar{y} = 1\;unit}$$

.

Centroid: $(2, 1)$

4.

$$A = \int y dx$$

$$= \int_0^\pi sinxdx$$

$$= [-cosx]$$

$$A = 2$$

$Mx = \int ycda; yc = \frac{y}{2}$

$= \int_0^\pi (\frac{y}{2}) ydx$

$= \frac{1}{2}\int_0^\pi y^2 dx$

$= \frac{1}{2}\int_0^\pi (sin^2 x) dx$

$= \frac{1}{2}\int_0^\pi (\frac{1-cos\,2x}{2})\,dx$

$= \frac{1}{2}(\frac{x}{2} - (2)\left(\frac{sin\,2x}{2}\right)dx$

$= \frac{1}{2}(\frac{x}{2} - \frac{sin\,2x}{4})$

$Mx = \frac{\pi}{4}(2)$

$\bar{x} = \frac{\pi}{2}$

$My = \int_0^\pi xcda; xc = x$

$= \int_0^\pi xydA$

$= \int_0^\pi xsinxdx$

$u = x \; ; \; dv = sinx$

$du = dx \; ; \; v = -cosx$

$= -cosx - \int -cosxdx$

$= [-xcosx + sinx]$

$= -\pi cos\pi + sin\pi + 0 - sin0$

$= \pi$

$\bar{y} = (\frac{\pi}{4})(2)$

$= \frac{\pi}{8}$

Centroid : $\left(\frac{\pi}{2}, \frac{\pi}{8}\right)$

5. F_x = 725 N · cos 37 º = 579.0 N $\quad$ r_x = 3.5 m

F_y = – 725 NN · sin 37 º = –436.3 N $\quad$ r_y = 6.0 m

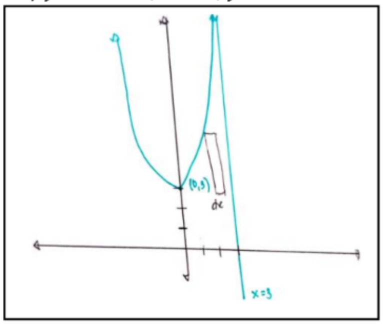
$M_{Ox} = F_x \cdot r_y \cdot \sin 90º$ = 579.0 N · 6.0 m = 3474 N · m

$M_{Oy} = F_y \cdot r_x \cdot \sin(-90º)$ = –436.3 N · 3.5 m · (–1) = 1527.05 N · m

M O = M Ox + M Oy = 5001.05 N · m perpendicular to the plane and in a clockwise direction.

CENTROID OF SOLID REVOLUTION

6. Determine the centroid of a solid generated by revolving the region bound by $y = x^4 + 3$, $x = 0$, $y = 0$, and $x = 3$ about the x-axis.



$= x^4 + 3$, $x = 0$, $y = 0$, and $x = 3$

Mass

$m = \int_0^3 (x^4+3)^2 dx$

$= \int_0^3 x^8 + 6x^4 + 9\, dx$

$= \frac{x^9}{9} + \frac{6x^5}{5} + 9x\Big|_0^3$

$= \left[\frac{(3)^9}{9} + \frac{6(3)^5}{5} + 9(3)\right] - \left[\frac{(0)^9}{9} + \frac{6(0)^5}{5} + 9(0)\right]$

$= \frac{12528}{5}$

$My = \int_0^3 x\,(x^4+3)^2 dx$

$= \int_0^3 x(x^8 + 6x^4 + 9)\, dx$

$= \int_0^3 x^9 + 6x^5 + 9x\, dx$

$= \frac{x^{10}}{10} + \frac{6x^6}{6} + \frac{9x^2}{2}\Big|_0^3$

$= \left[\frac{(3)^{10}}{10} + \frac{6(3)^6}{6} + \frac{9(3)^2}{2}\right] - \left[\frac{(0)^{10}}{10} + \frac{6(0)^6}{6} + \frac{9(0)^2}{2}\right]$

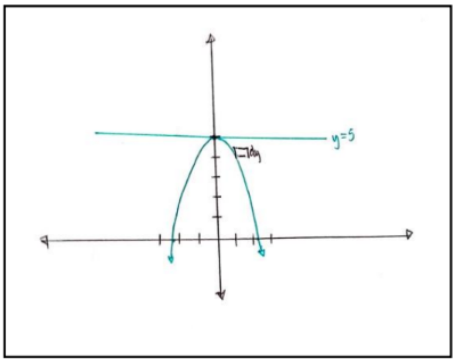
$= \frac{33372}{5} \div \frac{12528}{5}$

$= \frac{309}{116}$

Centroid

$(\bar{x}, \bar{y}) = \left(\frac{309}{116}, 0\right)$

7. Determine the centroid of a solid generated by revolving the region bound by $y = 5 - x^2$, $x = 0$ and $y = 0$ about the y-axis.



$y = 5 - x^2$

$x^2 = 5 - y$

$x = \sqrt{5-y}$

$x = \sqrt{5-y}$

$y = 5 - x^2$

$y = 5 - (0)^2$

$y = 5$

$a = 0, b = 5$

Mass

$m = \int_0^5 (\sqrt{5-y})^2\, dy$

$= \int_0^5 5 - y\, dy$

$= 5y - \frac{y^2}{2}\Big|_0^5$

$= \left[5(5) - \frac{(5)^2}{2}\right] - \left[5(0) - \frac{(0)^2}{2}\right]$

$= \frac{25}{2}$

$Mx = \int_0^5 y(\sqrt{5-y})^2\, dy$

$= \int_0^5 y(5-y)\, dy$

$= \int_0^5 5y - y^2\, dy$

$= \frac{5y^2}{2} - \frac{y^3}{3}\Big|_0^5$

$= \left[\frac{5(5)^2}{2} - \frac{(5)^3}{3}\right] - \left[\frac{5(0)^2}{2} - \frac{(0)^3}{3}\right]$

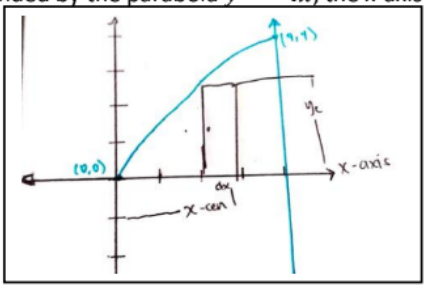
$= \frac{125}{6} \div \frac{25}{2}$

Centroid

$(\bar{x}, \bar{y}) = \left(0, \frac{5}{3}\right)$

$= \frac{5}{3}$

8. Find the centroid of the area bounded by the parabola $y^2 = 4x$, the x-axis and the line x=4.



$y^2 = 4x$ $\qquad x = 4$

If $x = 0$ $\qquad y = 0$

$x = 4$ $\qquad y = \pm 4$

$A = \int_0^4 y_c dx$

$= \int_0^4 2x^{\frac{1}{2}} dx$

$= 2(\frac{2}{3})(x^{\frac{3}{2}})\Big|_0^4$

$= \left[2(\frac{2}{3})(4^{\frac{3}{2}})\right] - \left[2(\frac{2}{3})((0)^{\frac{3}{2}})\right]$

$= \frac{32}{3}\ sq.units$

$A\bar{x} = \int x_{cen}(y_c)dA$

$= \int_0^4 x(2x^{\frac{1}{2}})dx$

$= \int_0^4 2x^{\frac{3}{2}} dx$

$= 2(\frac{2}{5})(x^{\frac{5}{2}})\Big|_0^4$

$= \left[2(\frac{2}{5})(4^{\frac{5}{2}})\right] - \left[2(\frac{2}{5})((0)^{\frac{5}{2}})\right]$

$= \frac{128}{5} \div \frac{32}{3}$

$= \frac{12}{5}$

$A\bar{y} = \int y_{cen}(y_c)dA$

$= \int_0^4 \frac{1}{2} y_c(y_c)\, dx$

$= \frac{1}{2}\int_0^4 {y_c}^2\, dx$

$= \frac{1}{2}\int_0^4 4x\, dx$

$= \int_0^4 2x\, dx$

$= 2(\frac{x^2}{2})\Big|_0^4$

$= x^2|_0^4$

$= 4^2 - 0^2$

$= 16 \div \frac{32}{3}$

$= \frac{3}{2}$

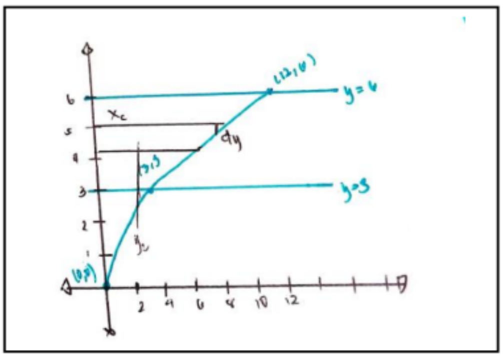
Centroid

$(\bar{x}, \bar{y}) = (\frac{12}{5}, \frac{3}{2})$

9. Find the centroid of the first quadrant region bounded by the curve $y^2 = 3x$, the y=axis and the lines y=3 and y=6.

$y^2 = 3x$

$x = \frac{y^2}{3}$



$A = \int_{y_1}^{y_2} (x_{right} - x_{left})\, dy$

$= \int_3^6 \frac{y^2}{3}\, \mathrm{dy}$

$= \frac{y^3}{3(3)}\Big|_3^6$

$= \left[\frac{(6)^3}{3(3)}\right] - \left[\frac{(3)^3}{3(3)}\right]$

$= 21$

$\bar{x} = \int_{y_1}^{y_2} x_c dA$

$= \int_3^6 \frac{\frac{y^2}{3}}{2}\left(\frac{y^2}{3}\right) dy$

$= \int_3^6 \frac{y^4}{18} dy$

$= \frac{y^5}{18(5)}\Big|_3^6$

$= \left[\frac{(6)^5}{18(5)}\right] - \left[\frac{(3)^5}{18(5)}\right]$

$= \frac{837}{10} \div 21$

$= \frac{279}{70}$

$\bar{Y} = \int_{y_1}^{y_2} y_c dA$

$= \int_3^6 y\left(\frac{y^2}{3}\right) dy$

$= \int_3^6 \frac{y^3}{3} dy$

$= \frac{y^4}{3(4)}\Big|_3^6$

$= \left[\frac{(6)^4}{3(4)}\right] - \left[\frac{(3)^4}{3(4)}\right]$

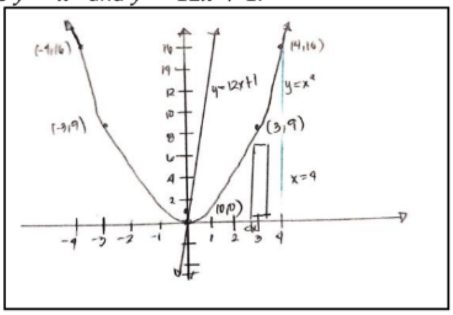
$= \frac{405}{4} \div 21$

$= \frac{135}{28}$

Centroid

$(\bar{x}, \bar{y}) = (\frac{279}{70}, \frac{135}{28})$

10. Find the centroid of the region bounded by the curves $y = x^2$ and $y = 12x + 1$.



$x^2 = 12x + 1$

$x^2 - 12x - 1 = 0$

$(x + 3)(x - 4)$

$x = -3 \qquad x = 4$

$(-3, 9)\ (4, 16)$

$y = x^2$

$y = (-3)^2$

$y = 9$

$y = x^2$

$y = (4)^2$

$y = 16$

$A = \int_{x_1}^{x_2} (y_{upper} - y_{inner}) dx$

$= \int_{-3}^{4} ((12x + 1) - x^2)\, dx$

$= \int_{-3}^{4} (-x^2 + 12x + 1)\, dx$

$= -\frac{x^3}{3} + 12\frac{x^2}{2} + x\Big|_{-3}^{4}$

$= -\frac{x^3}{3} + 6x^2 + x\Big|_{-3}^{4}$

$= \left[-\frac{(4)^3}{3} + 6(4)^2 + (4)\right] - \left[-\frac{(-3)^3}{3} + 6(-3)^2 + (-3)\right]$

$= \frac{56}{3}$

$A\bar{x} = \int_{x_1}^{x_2} x_c dA$

$\bar{x} = \int_{-3}^{4} x(-x^2 + 12x + 1) dx$

$\bar{x} = \int_{-3}^{4} (-x^3 + 12x^2 + 1x) dx$

$= -\frac{x^4}{4} + 12\frac{x^3}{3} + \frac{x^2}{2}\Big|_{-3}^{4}$

$= -\frac{x^4}{4} + 4x^3 + \frac{x^2}{2}\Big|_{-3}^{4}$

$= \left[-\frac{(4)^4}{4} + 4(4)^3 + \frac{(4)^2}{2}\right] - \left[-\frac{(-3)^4}{4} + 4(-3)^3 + \frac{(-3)^2}{2}\right]$

$= \frac{1295}{4} \div \frac{56}{3}$

$\bar{x} = \frac{555}{32}$

$A\bar{y} = \int_{x_1}^{x_2} y_c dA$

$\bar{y} = \int_{-3}^{4} \frac{1}{2}((12x + 1) + x^2)(-x^2 + 12x + 1) dx$

$= \frac{1}{2}\int_{-3}^{4} [(12x + 1)^2 - (x^2)^2] dx$

$= \frac{1}{2}\int_{-3}^{4} [144x^2 + 24x + 1 - x^4] dx$

$= \frac{1}{2}\left(\frac{144x^3}{3} + \frac{24x^2}{2} + x - \frac{x^5}{5}\right)\Big|_{-3}^{4}$

$= \frac{1}{2}\left(\frac{144x^3}{3} + 12x^2 + x - \frac{x^5}{5}\right)\Big|_{-3}^{4}$

$= \left[\frac{1}{2}\left(\frac{144(4)^3}{3} + 12(4)^2 + (4) - \frac{(4)^5}{5}\right)\right] - \left[\frac{1}{2}\left(\frac{144(-3)^3}{3} + 12(-3)^2 + (-3) - \frac{(-3)^5}{5}\right)\right]$

$= \frac{10514}{5} \div \frac{56}{3}$

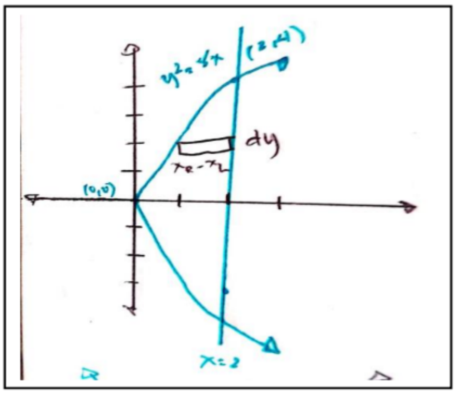
$= \frac{2253}{20}$

Centroid

$(\bar{x}, \bar{y}) = \left(\frac{555}{32}, \frac{2253}{20}\right)$

MOMENT OF INERTIA OF PLANE FIGURES

6. What is the moment of inertia of the area in the first quadrant bounded by the curves $y^2 = 8x$, x-axis and the line $x = 2$ with respect to the x-axis?



$y^2 = 8x \qquad x = 2 \qquad y^2 = 8x$

$I_x = \int y^2(x_r - x_l)dy \qquad y^2 = 8(2)$

$I_x = \int y^2\left(2 - \frac{y^2}{8}\right)dy \qquad y = 4$

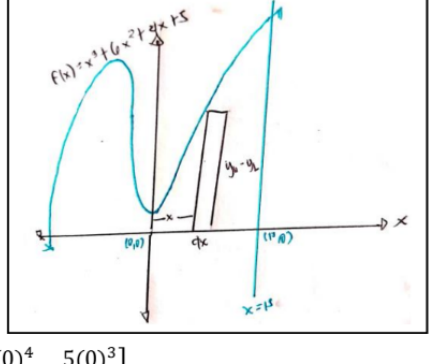
$$I_x = \int_0^4 2y^2 - \frac{y^4}{8}\, dy$$

$= \frac{2y^3}{3} - \frac{y^5}{8(5)}\Big|_0^4$

$= \left[\frac{2(4)^3}{3} - \frac{(4)^5}{8(5)}\right] - \left[\frac{2(0)^3}{3} - \frac{(0)^5}{8(5)}\right]$

$= \frac{256}{15}\ or\ 17.06667$

7. Find the moment of inertia of the area bounded by $y = 0, f(x), x = 0$ and $x = 15, f(x) = x^3 + 6x^2 - 4x + 5$ with respect to the y-axis.



$f(x) = x^3 + 6x^2 - 4x + 5 \qquad x = 0 \qquad x = 15$

$$I_y = \int x^2(y)dx$$

$$I_y = \int_0^{15} x^2(x^3 + 6x^2 - 4x + 5)\, dx$$

$I_y = \int_0^{15}(x^5 + 6x^4 - 4x^3 + 5x^2)\, dx$

$I_y = \frac{x^6}{6} + \frac{6x^5}{5} - \frac{4x^4}{4} + \frac{5x^3}{3}\Big|_0^{15}$

$= \left[\frac{(15)^6}{6} + \frac{6(15)^5}{5} - \frac{4(15)^4}{4} + \frac{5(15)^3}{3}\right] - \left[\frac{0^6}{6} + \frac{6(0)^5}{5} - \frac{4(0)^4}{4} + \frac{5(0)^3}{3}\right]$

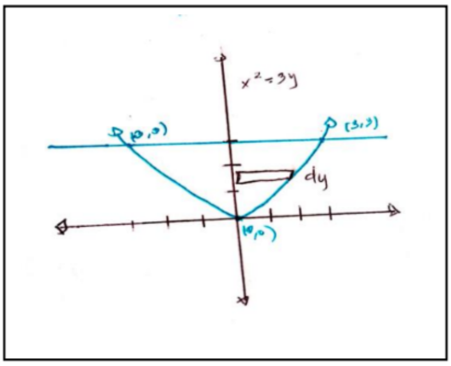
$= \frac{5529375}{2}\ or\ 2.8 \times 10^6$

8. Find the moment of inertia of the area bounded by the curve $x^2 = 3y$, the line $y = 3$ and the y-axis on the first quadrant with respect to the x-axis.

$x^2 = 3y$ $\quad$ $x^2 = 3y$ $\quad$ $y = 3$

$y = -\frac{x^2}{3}$ $\quad$ $x^2 = 3(3)$

$x = 3$



$$I_y = \int x^2(f(x))dx$$

$$= \int_0^3 x^2\left(3 - \frac{x^2}{3}\right)dx$$

$$= \int_0^3 \left(3x^2 - \frac{x^4}{3}\right)dx$$

$$= \frac{3x^3}{3} - \frac{x^5}{3(5)}\Big|_0^3$$

$$= \left[\frac{3(3)^3}{3} - \frac{(3)^5}{3(5)}\right] - \left[\frac{3(0)^3}{3} - \frac{(0)^5}{3(5)}\right]$$

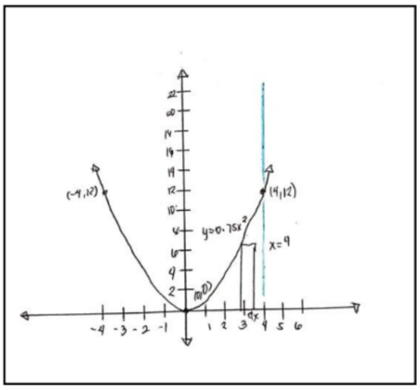
$$= \frac{54}{5}\ or 10.8$$

9. Determine the moment of inertia about the x-axis and the y-axis of the area bounded by $y = 0.75x^2$, the x-axis, and $x = 4$.

$y = 0.75x^2$ $\quad$ $x = 4$ $\quad$ $y = 0.75x^2$

$y = 0.75(4)^2$ $\quad$ $x^2 = -\frac{y}{0.75}$

$y = 12$ $\quad$ $x = \left(-\frac{y}{0.75}\right)^{\frac{1}{2}}$



$I_x = \int y^2(x_r - x_l)dy$

$= \int_0^{12} y^2\left(4 - \left(\frac{y}{0.75}\right)^{\frac{1}{2}}\right)dy$

$= \int_0^{12}\left(4y^2 - \frac{2y^{\frac{5}{2}}}{\sqrt{3}}\right)dy$

$= \frac{4y^3}{3} - \frac{4y^{\frac{7}{2}}}{7\sqrt{3}}\Big|_0^{12}$

$= \left[\frac{4(12)^3}{3} - \frac{4(12)^{\frac{7}{2}}}{7\sqrt{3}}\right] - \left[\frac{4(0)^3}{3} - \frac{4(0)^{\frac{7}{2}}}{7\sqrt{3}}\right]$

$= \frac{2304}{7}\ or\ 329.143$

$I_y = \int x^2(y)dx$

$= \int_0^4 x^2(\,0.75x^2 - 0)dx$

$= \int_0^4 \frac{3x^4}{4}dx$

$= \frac{3x^5}{4(5)}\Big|_0^4$

$= \left[\frac{3(4)^5}{20}\right] - \left[\frac{3(0)^5}{20}\right]$

$= \frac{768}{5}\ or\ 153.6$

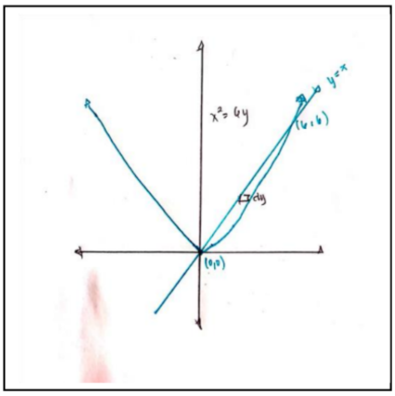
10. Determine the moment of inertia about the x-axis and the y-axis of the area bounded by $x^2 = 6y$, and the $y = x$.

$x^2 = 6y$ $\quad$ $y = x$ $\quad$ $x^2 = 6y$

$y^2 = 6y$ $\quad$ $x = 6$

$y = 6$

$PI(6,6)$



$I_x = \int y_c{}^2 dy$

$= \int_0^6 y^2\left((6y)^{\frac{1}{2}} - y\right)dy$

$I_y = \int x_c{}^2 dy$

$= \int_0^6 x^2\left(x - \frac{x^2}{6}\right)dx$

$$= \int_0^6 \sqrt{6}\, y^{\frac{5}{2}} - y^3 dy$$

$$= \frac{2\sqrt{6}y^{\frac{7}{2}}}{7} - \frac{y^4}{4}\Bigg|_0^6$$

$$= \left[\frac{2\sqrt{6}(6)^{\frac{7}{2}}}{7} - \frac{(6)^4}{4}\right] - \left[\frac{2\sqrt{6}(0)^{\frac{7}{2}}}{7} - \frac{(0)^4}{4}\right]$$

$$= \frac{324}{7}\ 0r\ 46.286$$

$$= \int_0^6 \left(x^3 - \frac{x^4}{6}\right) dx$$

$$= \frac{x^4}{4} - \frac{x^5}{6(5)}\Bigg|_0^6$$

$$= \left[\frac{x^4}{4} - \frac{x^5}{6(5)}\right] - \left[\frac{x^4}{4} - \frac{x^5}{6(5)}\right]$$

$$= \frac{324}{5}\ or\ 64.8$$